Study of Changes in the Production Process for Short-Life Cycle Goods

Abstract
Production management has to react to changing exogenous and endogenous factors. A method called the Production Process Control Tree (PPCT) was developed to control the production process and analyse its changes. The method’s name refers to its underlying production management model, which is shaped as an inverted tree (graph theory). The method helps to cushion or even eliminate the negative impacts of changing process circumstances that could extend their duration.

Key words: short-life cycle, production process, production management.

Introduction
According to research, apparel as a category of short life-cycle goods is made in a production process (consisting of production creation, the setting up of the manufacturing processes, the procurement of intermediate materials and sale) that spans 18 months [1]. In this relatively long time, the actual production circumstances and those predicted at the planning stage are very likely to show significant differences. Why the changes appear will not be explored in the article to keep it concise, but their occurrence will be assumed a priori as fact. This study was designed to investigate the negative effects of changes delaying the completion of a production process.

It must be remembered that the production cycle for short life-cycle goods has a pre-determined date when the sale should start. The fact that the delays observed during the research usually resulted from management errors committed due to the lack of an appropriate tool aiding production management helped identify a cognitive gap, following which a method for analysing process changes was developed. The method was called the Production Process Control Tree (PPCT).

A critical path and indicator of process resistance in the production management model

The critical path of the tree determines the duration of the production process, in the same way as a network’s critical path does. By identifying the critical path of the production management model and by calculating the Indicator of Process Resistance (IPR) to change, the manager can estimate the degree to which the process completion date is at risk.

Let us present a procedure for finding a critical path or paths for the production management model that do not offer time reserves. Their number provides an indication of the process resistance to changes. The tree has at least one critical path, but every path running between an initial state and a final state can theoretically be a critical one. Then the production process is likely to end later than scheduled, because of changes increasing the duration of any of its constituent tasks. Production processes based on management models with a large number of critical paths are not resistant. The most resistant are processes having only one critical path. This shows that a production management model should be analysed while still being planned, as the range of model verification options is the greatest then.

Let us present a procedure that was developed to identify the critical path of a production management model shaped as a rooted tree. Let vertex \( w \) correspond to the tree’s root (a final event of the production process). Among the states directly preceding state \( w \) there is a state \( u \) with a time reserve \( t(u) = 0 \). If all states coming before state \( w \) had time reserves, then state \( w \) would also have a time reserve. This contradicts the assumption on which model [2] was founded, i.e.:

\[
p(w) = q(w), \text{ i.e. } r(w) = 0
\]

Applying the same reasoning to state \( u \) and its preceding states, we infer that there is a state \( v \) preceding state \( u \), for which \( r(v) = 0 \). This procedure should be repeated until we discover that a state without a time reserve is one of the initial states (let us call it vertex \( a \)). The path starting at \( a \) and ending at \( w \) will be called the critical path. Accordingly, each state on the critical path is a critical state, particularly the initial state \( a \).

While trying to identify the critical path, we may discover that the critical state analysed \( u \) is preceded by more than one critical state, in which case more than one critical path goes through state \( u \). This means that the production management model (for short life-cycle products) has as many critical paths as critical initial states. Let us suppose that the model has \( n \) initial states containing \( k \) critical states. According to earlier observations, the value of \( k \) is within the range:

\[
1 \leq k \leq n
\]

Consequently, the Indicator of Process Resistance to unexpected changes can be calculated as follows:

\[
IPR = \frac{n - k}{n} = 1 - \frac{k}{n}
\]

where:
- \( IPR \) – Indicator of Process Resistance to unexpected changes,
- \( n \) – the number of initial states (tree leaves),
- \( k \) – the number of critical states among the initial states.

IPR’s extreme values are obtained for \( k = n \) and \( k = 1 \), which define the range of values for the indicator created, i.e.:

\[
< 0; 1 - 1/n >
\]

When every path in the production management model is critical, then \( k = n \) and the IPR = 0. In the model with a single critical path \( k = 1 \), and the IPR is close to one because it is calculated as:

\[
IPR = 1 - 1/n \approx 1
\]

It can be assumed that the IPR value thus defined measures the process resistance to changes that delay its completion. The indicator can be used for assessing the production management model’s design, as well as its actual performance (each time it has been modified). When the IPR is 0, then the timely completion of the production process is
very much at risk, should any change extending the duration of any of its tasks occur. This means that the IPR is a synthetic measure of process resistance to changes, regardless of the stage they affect.

The PPCT as a method of investigating changes in the production process

A production management model (an inverted tree) provides information on the tasks, their duration and relationships. This aggregate knowledge allowed to develop a method that:
- controls the duration of process-related tasks,
- monitors changes affecting task duration,
- allows to adjust the model when the changes delay the end date of the production process.

Let us present the Production Process Control Tree (PPCT) method, which has been specifically developed for short-life cycle goods. Let us note that the occurrence of state \( u \) can be delayed with respect to the time \( p(u) \), because of one of two reasons:
- one of the states directly or indirectly preceding state \( u \), e.g. \( a \), occurs later than \( p(a) \),
- one of the tasks preceding state \( u \), e.g. \( a-u \), stretches over a longer period than the scheduled time \( t(a-u) \).

where:
- \( p(u) \) – the earliest moment of commencing the task originating in vertex \( u \),
- \( p(a) \) – the earliest moment of commencing the task originating in vertex \( a \),
- \( t(a-u) \) – the time of performing the task between vertices \( a \) and \( u \).

The delayed occurrence of state \( u \) may make shift the end of production to a later date. This is certain to happen when \( u \) is a critical state. In general a delay will only take place when the delayed occurrence of state \( u \) affects the commencement of the nearest critical state situated on the path linking state \( u \) and the tree root. However, the delay of state \( u \) may also be ‘absorbed’ by the time reserved of the states following \( u \).

For the production management model to monitor changes, we need to find times \( p(u) \) and \( q(u) \), representing the earliest moment of commencing each task and the latest moment of ending each task, respectively. According to the literature, the following rules can be applied to find the duration of the tasks:
- a deterministic rule for tasks carried out according to the company’s own rules that explicitly prescribe the task’s deadline or duration,
- a probabilistic rule for tasks of duration determined empirically by an experienced expert.

Knowledge of the rules for determining the duration of tasks and time reserves played an important role in developing the PPCT method. Each time reserve indicates the length of time by which a task can extend without the production management model having to undergo adjustment. Considering that a production process is shaped by many variables, the model constructed has to be dynamic, even enabling some modifications to the schedule in case the end date of the production process becomes uncertain.

Let us suppose that we have a production management model shaped as an inverted tree. The duration of particular tasks and the moments when the initial states of the tasks should start are also known. The duration of the task transforming state \( a \) into \( b \) is denoted as \( t(a-b) \), and the earliest moment when state \( u \) can commence is denoted as \( p(u) \). Let us create an algorithm for this production management model to find \( p(a_i) \), \( q(w) \) and \( q(a) \), the values of which will be used when the production management model has to account for the impacts of variables affecting the production process. Hence:
- \( p(a_i) \) is the earliest moment allowed when the task originating in vertex \( a_i \) should commence; it will be calculated according to formula (1) below,
- \( q(w) \) is the latest permissible moment of ending the final process activity \( w \); it will be calculated according to formula (2) below,
- \( q(a) \) is the latest time allowed when the task originating in vertex \( a \) should end; it will be calculated according to formula (3) below.

Let us use this method for calculating the time \( p(u) \) for vertex \( u \), which is not a tree leaf (i.e. an initial state). In the tree considered, the offspring of vertex \( u \) are the vertices that come immediately before it. Then:

\[
p(u) = \max \{ p(a_i) + t(a_i - u) \} \quad \text{for } i = 1, \ldots, k
\]

where:
- \( p(u) \) – the earliest moment when the task originating in vertex \( u \) should commence;
- \( p(a_i) \) – the earliest moment when the task originating in vertex \( a_i \) should commence,
- \( t(a_i - u) \) – the duration of the task between vertices \( a_i \) and \( u \).

The rule above helps to find the moments \( p(w) \), first for the states directly following the initial states and then recurrently for all vertices of the tree representing the production management model.

Let \( w \) be a state equivalent to the tree root and \( p(w) \) the actual end date of the production process. If \( T \) is the scheduled end date, then the process will end as planned when:

\[
p(w) \leq T
\]

If otherwise, the product will not be ready on time. This problem can be handled by adjusting the process, which entails some restructuring of the production management model. Continuing the earlier procedure, we determine the moment \( q(w) \), i.e. the latest time allowed when state \( u \) should commence. Naturally, the relationship:

\[
q(w) = T
\]

still holds.

Building on the earlier assumption about the management of the production of short life-cycle goods, we can state that:

\[
p(w) = q(w) = T
\]

where:
- \( p(w) \) – the earliest moment allowed when the task originating in vertex \( w \) (the tree root) should commence,
- \( q(w) \) – the latest moment allowed when task \( w \), being the final task of the production process, should end,
- \( T \) – the end of the scheduled process.

If the equality \( p(w) = q(w) = T \) does not take place, then condition (2) can be met by introducing a dummy state \( w \); assuming that:

\[
p(w') = T \quad \text{and} \quad (w' - w) = T - p(w)
\]

The time \( t(w' - w) \) shows the shift in the product completion date, thus providing information on the effect the changes observed have on the production process. According to assumption (2), vertex \( w \), ending the production process (i.e. the tree root), is a critical state. Thus
the moment \( q(u) \) of state \( u \) is determined, and state \( a \) precedes state \( u \) towards the root. Then \( q(a) \) can be defined using the following equation:

\[
q(a) = q(u) - t(a - u) \tag{3}
\]

where:

- \( q(a) \) – the latest moment allowed when the task originating in vertex \( a \) preceding vertex \( u \) should end,
- \( q(u) \) – the latest moment allowed when the task originating in vertex \( u \) should end,
- \( t(a - u) \) – the duration of the task between vertices \( a \) and \( u \).

The value on the right hand-side of equation (3) is determined precisely as there is only one edge that starts at \( a \) and ends at \( u \). Consequently, the formula needs neither a minimum nor a maximum operator. Formula (3) allows the recurrent determination of the time \( q(u) \) for each vertex of the tree, whose property is utilized in the PPCT method to monitor changes.

Changing production circumstances may extend the amount of time needed to end the process. This threat must result in an immediate correction of the model data. In other words, changing circumstances should generate warnings about a possibly delayed product completion date.

### The PPCT method

We propose for investigating production changes compares the duration of each task with its scheduled time. Let us assume that a task \( a - x \) was performed in time \( t'(a - x) \), which extended beyond its scheduled time \( t(a - x) \) by \( n \) units. Then we have:

\[
t'(a - x) = t(a - x) + n
\]

Let us also assume that the path linking state \( x \) and the tree root successively goes through states \( y_1, y_2, \ldots, y_k \), hence the path is given as \( x - y_1 - \ldots - y_k - w \). We additionally assume that all tasks preceding state \( a \) were performed on time, meaning that activity \( a - x \) started at the moment \( p(a) \). In this case, one of the following three possibilities takes place:

1. \( p(a) + t(a - x) + n \leq q(x) \), or
2. \( p(x) < p(a) + t(a - x) + n \leq q(x) \), or
3. \( q(x) < p(a) + t(a - x) + n \).

where:

- \( p(x) \) – the earliest moment when the task originating in vertex \( x \) should commence,
- \( q(x) \) – the latest moment when the task originating in vertex \( x \) should end,
- \( p(a) \) – the earliest moment when the task originating in vertex \( a \) preceding state \( x \) should commence
- \( t(a - x) \) – the duration of the task between vertices \( a \) and \( x \),
- \( n \) – the number of units by which the duration of task \( a - x \) has been extended.

Let us now explore the meaning of the three situations and find the algorithms to deal with them.

Should the first case occur, the end date of the production process runs no risk of being delayed, because the earliest moment when task \( y_1 \) (initiation of activity \( x \)) should start takes place after the length of time allocated to activity \( a - x \) elapses. Hence the model of the process does not need any modifications, except replacing time \( t(a - x) \) by \( t'(a - x) = t(a - x) + n \). In this case, the production manager does not have to be informed about an event if it occurs, as its influence is neutral.

In the second case, the production process is not exposed to any direct threat to its timely completion because state \( x \) has a time reserve \( r(x) = q(x) - p(x) \). Task \( a - x \) will end not later than \( q(x) \), being the latest moment allowed for task \( x - y_1 \) to commence. In this situation, the tree requires the following modifications:

1. Time \( t(a - x) \) has to be replaced with \( t'(a - x) = t(a - x) + n \).
2. Each state on the path \( x - y_1 - \ldots - y_k - w \) has to be assigned a new commencement time using formula (1), and some obvious changes have to be made to the formula’s symbols (\( u \) has to be replaced with the name of the right vertex and \( a \) with its preceding vertices).

In the third case, the end date of the production process will be exceeded because state \( x \) needs more time to end than its time reserve \( r(x) \) allows. Hence, the model has to be modified as follows:

(i) the time \( t(a - x) \) has to be replaced with \( t'(a - x) = t(a - x) + n \),
(ii) each state on path \( x - y_1 - \ldots - y_k - w \) has to be assigned a new commencement time using formula (1), and some obvious changes have to be made to the formula’s symbols (\( u \) has to be replaced with the name of the right vertex and \( a \) with its preceding vertices),
(iii) the process end date \( T \) has to be replaced with \( T' = p(w) \), assuming at the same time that \( q(w) = p(w) \), where the time \( p(w) \) represents a new process end date calculated at step (ii),
(iv) now the latest moments of starting tasks allowed that have not been carried out yet have to be determined, the duration of the tasks remaining the same.

Steps (i) ÷ (iv) adjust the model of the process. The production manager has to be notified of the situation to decide on the next steps after analysing the new model. The manager may choose to shorten the sequence of activities \( x - y_1 - \ldots - y_k - w \) by introducing organisational, technical or technological improvements. If the intervention is effective and, for instance, the time \( t(y_1 - y_i+1) \) becomes shorter by \( m \) units, then steps (i) ÷ (iv) should be repeated, with the time \( t(y_i - y_{i+1}) \) at step (i) being replaced by \( t'(y_i - y_{i+1}) = t(y_i - y_{i+1}) - m \).

Because decisions on taking actions that cause dynamic adjustment of the model usually increase product manufacturing costs, the company board has to grant the production manager an appropriate scope of authority. Otherwise, the PPCT method, enabling interactive management of production processes, will not be as effective as it can be. If the duration of the longest path of the tree ensures following an intervention that makes sure the production process will end as scheduled, then the process is continued. If otherwise, the production manager has to notify the Board (or another relevant body) of the situation, which may choose to discontinue the production process (after estimating the losses) or to carry on.
Conclusions

All computations (necessary to design the original tree, to determine its longest path and readjust the production management model) can be performed in real time, once appropriate computer software has been developed. The need for the management process to integrate technology, organisational issues and IT, regardless of the supervisory, stimulating role of the process manager, was accentuated by B. Nogalski [3]. Software should be built around the aforementioned algorithm that enables study of changes in the production of short life-cycle goods. Such software helps implement the method we propose in business practice. Making decisions under the pressure of time is a key problem that companies manufacturing short life-cycle products have to resolve. Modern production management methods, including the PPCT, combined with IT tools are the only ones that make it possible to:

- analyse a production process and its changing circumstances on an on-going basis,
- make decisions in real time to offset the negative impacts of changing process circumstances,
- minimise the losses a company may incur should it decide to discontinue the production process, as every step forwards generates unnecessary costs (augmenting the losses).

The method proposed for analysing changes in the production process can improve the effectiveness of companies making short life-cycle goods that function in turbulent environments. According to the Global Trends 2025 report prepared by the National Intelligence Council, such environments are becoming the norm today [4]. The report reveals not only problems but also opportunities that arise from unexpected changes creating new economic realities.

References