Optimisation of the Knitting Process on Warp-Knitting Machines in the Aspect of the Properties of Modified Threads and the Vibration Frequency of the Feeding System

Abstract

Important factors determining the behaviour of the knitting process on warp knitting machines are the mechanical properties of warp threads. This is caused by the variety of the properties of threads, ranging from "rigid" threads, such as steel, aramid, basalt, and glass threads intended for technical products to "elastic" threads in the form of elastomers, intended for linen and clothing products. Great differences in the rigidity and viscosity of threads significantly influence the parameters of the knitting process. As a result of the simulations performed, it was noted that there is a correlation between the elasticity, the damping of the feeding system, the dynamic loads of threads, the vibration frequency of the feeding system and the rheological parameters of threads. In conclusion, with an increase in the rigidity of threads, the elasticity of the system and frequency of vibrations in the system increase as well. The above-mentioned parameters of the process that are dependent on the coefficient of thread damping change in an inversely proportional manner.

Key words: core-spun yarn, coating of core part, strip-back, abrasion resistance, pilling.

by a linear differential equation of the second order, in the following form [2]:

\[ m \frac{d^2 y}{dt^2} + b_{2\alpha} \frac{dy}{dt} + k_{2\alpha} y = a_i \left[ k_p \cdot S(t) + b_p \frac{dS(t)}{dt} \right] \] (1a)

or in other form:

\[ \frac{d^2 y}{dt^2} + 2h \frac{dy}{dt} + \omega_n^2 y = \frac{a_i}{m} \left[ k_p \cdot S(t) + b_p \frac{dS(t)}{dt} \right] \] (1b)

where:

- \( m \) – point mass of the tension rail reduced to one thread
- \( b_{2\alpha} \) – coefficient of system attenuation
- \( k_{2\alpha} \) – coefficient of system elasticity
- \( a_i \) – coefficient describing the geometry of the feeding system
- \( h \) – relative coefficient of system attenuation
- \( \omega_n \) – frequency of free vibration of the non-attenuated system
- \( k_p \) – coefficient of thread elasticity
- \( b_p \) – coefficient of thread attenuation
- \( S(t) \) – kinematic input function of the feeding system.

The equations describing the deflection of the tension rail \( y(t) \) and the dynamic forces in the threads \( P(t) \) feeding the knitting zone [1] are solutions of the mathematical model. A computer program simulating the knitting process on warp-knitting machines, described as a function of input parameters of the process, was elaborated [6]. The aim of research presented in this article was to analyse the loads of threads and the frequency of vibrations of the feeding system with regard to mechanical properties of threads treated as visco-elastic objects, as described by the Kelvin-Voight model.

Analysis of the elasticity and attenuation of the feeding system with regard to the character of changes in the reduced rigidity, attenuation coefficients and optimisation of forces in threads

Analysis of the feeding system’s elasticity

The coefficient of system elasticity \( k_{stri} \), presented as the product of tension rail displacement \( y \) in the equation of motion (1) of the feeding system, determines the component of the dynamic force – the elasticity force.

Coefficient \( k_{stri} \), described by formula

\[ k_{stri} = k_p = \frac{k_p}{\alpha + a \cdot (\cos \beta - \sin \alpha) \cdot k_p}, \] (2)

(where \( k_p \) – rigidity coefficient of tension rail, reduced to one thread, \( a \) and \( \beta \) – angles of the thread „running-on” and „running-off” the tension rail), is a function of the rigidity coefficient of the tension rail \( k_p \), the coefficient of thread elasticity \( k_p \) as well as the coefficient describing the geometry of the system \( a \) and the difference between angles \( \alpha \) and \( \beta \) of trigonometric functions \( (\cos \beta - \sin \alpha) \).
For constant parameters of the thread’s feeding geometry, that is \(a_i(\cos \beta - \sin \alpha) = k = \text{const.}\), and assuming that the conditions of forcing the threads are constant, Equation 2 can be presented in the following form:

\[
k_z r_i = k_s + k_p (3).
\]

This dependence describes the linear character of function \(k_z r_i = f(k_s \text{ and } k_p)\). The plane, which is a graphical presentation of dependence (2), is a net of crossing parallel linear functions (Figure 1).

In view of the diversified structure of warp-knitting machines, the character and the values of elasticity \(k_z r_i\) should be considered in the aspect of the variation of the coefficient describing the geometry of feeding system \(a_i\) and angles \(\alpha \text{ and } \beta\). Figures 2 and 3 present graphs describing the dependencies of \(k_z r_i\) of function \(\alpha\) and \(\beta\). The field describing function \(k_z r_i = f(\alpha \text{ and } \beta)\) for \((\mu \text{ and } l_1) = \text{const}\) has a decreasing character within the range of a decreasing angle \(\alpha\) from -60 to +50°, while above 50° a slightly increasing tendency can be observed. As results from the graph \(k_z r_i = f(\alpha)\) for \(\beta = \text{const}\) presented (Figure 2), a decrease in the value of coefficient \(k_z r_i\) reliably describes a function of a polynomial of the third degree, depending on angle \(\alpha\), with correlation coefficients of boundary curves \(R^2 \geq 0.998\). Variation \(k_z r_i = f(\beta)\) (Figure 3) takes values symmetrical to OY (for \(\beta = 0\)), and it can be described by a parabolic equation in the form of \(k_z r_i = -a\beta^2 - b\beta + c\), although in reality it is a complex function describing the dependencies of trigonometric functions sinus and cosine, the power function and exponential function. The character of these changes is determined by that of changes in coefficient \(a_i\) and the following term \((\cos \beta - \sin \alpha)\).

Summing up the analysis of \(k_z r_i\) mentioned above, it can be stated that the coefficient of system elasticity depends in a linear manner (increasing) on the in-
creasing values of the rigidity of the tension rail springs \( k_t \) and on the elasticity of the thread \( k_p \) as well as on the parameters of the feeding system geometry.

**Analysis of the damping of the feeding system**

Coefficient \( b_{zr} \) is a coefficient of the linear dependence of the viscose damping force on the speed of tension rail dislocations of the feeding system: \( b_{zr} = dy/dt \). Coefficient \( b_{zr} \) describes the following dependence:

\[
b_{zr} = b_p (\cos \alpha - \sin \alpha) a_t \quad (4).
\]

In the simplified form

\[
b_{zr} = k_t \cdot b_p, \quad (5)
\]

and for a constant geometry of feeding

\[
k_t = a_p (\cos \beta - \sin \alpha) = \text{const.}
\]

The linear character of function \( b_{zr} = f(b_p) \) is presented in Figure 4, from which it results that the only (assumed) factor defining the damping of the feeding system from the point of view of the visco-elastic properties of the thread is the coefficient of thread attenuation \( b_p \).

**Equation 5** is also determined by the values of coefficient \( k_p \), which depends on the parameters of the feeding system geometry \( k_t = f(l_1, l_2, \mu, \alpha, \beta) \), where:

- \( l_1 \) — length of thread from the tension rail to the guide bar,
- \( l_2 \) — length of thread from the warp beam to the tension rail,
- \( \mu \) — coefficient of friction between the tension rail and thread. The character of function: \( b_{zr} = f(\alpha, \beta) \) is identical to the following dependence: \( k_{zr} = f(\alpha, \beta) \). The dependence \( k_p = f(\alpha, \beta) \) can be referred to the quality analysis of function: \( b_{zr} = f(\alpha, \beta) \).

The relative coefficient of system attenuation \( b_{zr} \) is described by the ratio of the coefficient of system attenuation \( b_{zr} \) to the mass of the tension rail reduced to one thread \( m \):

\[
h_i = b_{zr}/2m, \quad (6).
\]

The dependence of \( h_i \) on coefficient \( b_{zr} \) is determined by an increasing linear function in the following form: \( h_i = k \cdot b_{zr} \), in which proportionality factor \( k \) is determined by an increasing linear function in the following form:

\[
h_i = k \cdot b_{zr}, \quad (7).
\]

Computer simulation of the knitting process was performed to enable variation analysis of forces in threads considering the function of mechanical parameters of threads, that is the coefficient of elasticity and attenuation \( k_p \) and \( b_p \).

**Analysis of dynamic loads of threads with respect to their mechanical parameters**

Calculations were performed for input data with reference to the K2 (E20) warp-knitting machine made by K. Mayer.

**Figures 6 and 7** present the dependencies of \( P_{\text{max}} = f(k_p, b_p) \), from which it results that extreme forces in threads increase with their increasing elasticity and decreasing viscose damping.

This character of variation in forces relates to literature data [7, 8], which indicates that in the case of processing of the elastomer threads joined, textured, and characterised by low values of rigidity and high values of attenuation, the knitting process is performed at low values of forces in the threads and low values of thread breakage.

For ‘rigid’ threads, such as synthetic monofilaments PA & PE, steel monofilaments, aramid and glass threads, there are high values of forces in the feeding zone, two or three times bigger than the average values, being within the range from 20 to 40 cN and even more, which are the cause of frequent breakings. And in the case of threads of high values of tensile strength, damage to loop forming elements of the machine also occurs (breaking or permanent deformations).

The dependencies presented in Figures 6 and 7 confirm the correctness of the model while calculating the effect of forces acting at assumed mechanical (rheological) parameters.

**Analysis of the vibration frequency of the feeding system**

In the mathematical model elaborated, the free vibration frequency of the feeding system is determined by two quantities:

- \( \omega_0 \) as the free vibration frequency of the non-attenuated system, being a square root of a fraction of the coefficient of system elasticity \( k_{zr} \), referring to the reduced mass of the tension rail \( m \):

\[
\omega_0 = (k_{zr}/m)^{1/2}, \quad (7);
\]

- \( \alpha \) as the ratio of the force of breaking \( P_{\text{max}} \) and the breaking point distance \( l_{\text{max}} \), in which \( \alpha = P_{\text{max}}/l_{\text{max}} \) — the point of breaking.

In the mathematical model elaborated, the free vibration frequency of the feeding system is determined by two quantities:
ω as the free vibration frequency of the attenuated system, being a square root of the difference in the squares of frequency ω₀ and relative coefficient of attenuation h:

$$\omega = \left(\omega_0^2 - h^2\right)^{\frac{1}{2}} \quad (8).$$

Both frequencies ω and ω₀ are determined by the same input parameters of the process, such as the rigidity of the tension rail spring kₜ, the coefficient of thread elasticity kₚ, and the coefficient of thread attenuation bₚ. For the geometry of the system and mass of the tension rail given:

$$ω₀ = \left[kₜ + a \left(\cos β - \sin α\right) \cdot kₚ\right]^{-\frac{1}{2}} \cdot m^{-\frac{1}{2}} \quad (9)$$

$$ω = \left[kₜ + a \left(\cos β - \sin α\right) \cdot kₚ\right]^{-\frac{1}{2}}$$

$$= kₜ + a \left(\cos β - \sin α\right) \cdot kₚ + bₚ \left[\left(\cos β - \sin α\right) \cdot a / 2m\right]^2 \quad (10)$$

The free vibration frequency of the non-attenuated system ω₀ increases with increasing values of thread elasticity kₚ (system elasticity kₚ₀) and rigidity of the tension rail spring kₜ according to power function $y = ax^b$ with different values of coefficient $a = (1/m)^{\frac{1}{2}}$.

Figures 8 and 9 present the character of function $ω₀ = f(kₚ)$ and $ω₀₁ = f(kₚ₀)$. In Figure 9, within the whole range of $kₚ₀ = 0.0075 - 0.8421$ cN mm⁻¹, the graph is of the character of a power function, while in the real range (physically real) for $kₚ$, it can be described by the linear regression equation $y = 0.87x + 1.6$ for $R² = 0.9966$ or power equation $y = 1.72x^{0.572}$ for the coefficient of regression $R² = 0.9972$.

According to Equation 10, the free vibration frequency of the attenuated system ω is a function whose values increase with an increase in the elasticity of threads kₚ (coefficient of system elasticity kₚ₀) according to the linear or power function (depending on the range of the independent variable of regression assumed) for a different bₚ, and also a function whose values decrease with increasing values of thread attenuation bₚ (coefficient of system attenuation bₚ₀), described mathematically by equations of polynomial regression of the third degree for $R² ≤ 0.996$. The dependencies mentioned above are presented in Figure 10. In the case of the frequencies ω₀ and ω analysed, it should be noted that the frequencies of the vibration of the un-attenuated system are determined by the parameters of thread elasticity and tension rail rigidity, whereas in the case of vibration frequencies of the attenuated system, an additional factor determining this frequency is the viscous damping of the thread bₚ.

Figure 7. Dependence of force $P_{max}$ on the coefficient of thread attenuation.

Figure 8. Dependence of the vibration frequency $ω₀$ of the system on thread elasticity $kₚ$.

Figure 9. Dependence of the vibration frequency of the system $ω₀$ on the elasticity of the system $kₚ₀$.

Figure 10. Dependence of the free vibration frequency of the attenuated system $ω₁$ on the viscoelastic properties of threads $kₚ$ and $bₚ$. 
Conclusions

Simulation tests of the knitting process were performed on warp-knitting machines in the aspect of the optimisation of this process, using an analysis of the dependencies of forces in the threads on their mechanical properties. The analysis also described particular correlations of the free vibration frequency of the feeding system on the parameters of threads. The tests were performed in a "technological" aspect, that is investigating the relations of changes in the parameters of the knitting process on warp-knitting machines depending on the rheological properties of the threads modified, often ignoring research problems of the knitting process in the aspect of structural properties of machines presented in literature.

From the tests on the knitting process performed, it results that:

The elasticity coefficient of the feeding system determining the dynamic component (elasticity force) of mechanical vibrations increases linearly depending on the increase in rigidity of tension rail springs and the elasticity of threads. The coefficient also depends on the parameters of the feeding system geometry.

Attenuation of the feeding system using the threads of the knitting zone increases in a linear manner along with the viscosity coefficient of the threads, with the intensity of the increase depending on the mass of the tension rail and the parameters of the geometry of the feeding zone.

The free angular frequency of the feeding system increases (with high probability) in a linear manner with the elasticity of the thread and elasticity of the system, and decreases (polynomial of the third degree with $R^2 \leq 0.996$) with an increase in the coefficient of thread attenuation.

The dependencies of forces in threads determined by their mechanical parameters indicate a correct reaction of the model in the aspect of a cause-effect analysis. Forces in the threads increase with an increase in the coefficient of thread elasticity, whereas with an increase in the coefficient of the viscose damping of the thread, the forces decrease, which confirms the industrial practice of processing "rigid" threads, such as synthetic monofilaments, steel threads, glass and aramid threads, as well as "elastic" threads of the textured and elastomeric type.

The results obtained can be useful in the optimisation of the knitting process in the aspect of selecting mechanical parameters of threads with additional conditions, which can provide correct (minimum) loads of threads and lead to a state in which the frequencies of attenuated vibrations of the tension rail system will be out of the resonance range.

Acknowledgment

This work was (partially) supported by structural funds within the framework of the project entitled 'Development of a research infrastructure for innovative techniques and technologies of the textile clothing industry’ CLO – 2IN – TEX, financed by Operational Programme Innovative Economy, 2007-2013, Action 2.1.

References


Received 01.03.2011 Reviewed 23.05.2011