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Bending Rigidity of Yarn Using a Two Supports Beam System

Abstract

The simple cantilever is a common method to measure the bending rigidity of fabrics. However, the small dimension and untwisting of the free end of yarn are the major difficulties in the simple cantilever method. In this work, a two support beam system was used to measure the bending rigidity of yarn. The yarn was treated as an elastic beam fixed at one end, supported by a simple support at the other, and loaded near the middle. The maximum deflection of the yarn and the distance of the applied load from the supports were measured accurately. Using classic elastic equations in the small deflection case, the bending rigidity of the yarns was then calculated. Comparison between the shape of deflection of the yarn and the elastic beam curve in the small deflection case showed reasonable agreement. It was concluded that the bending rigidity of yarns can be calculated by using the small deflection equations.

Key words: *bending, bending rigidity, elastic beams, small deflection.*

Introduction

A variety of the practical end-use considerations of textile structures are associated with the bending rigidity of yarns comprising a structure. The association may be obvious or subtle, and in either case the magnitude of the interrelationship may be large or small. Examples of obvious associations are the flexural rigidity or drape ability of fabrics, and fabric crease resistance or resistance to bending. Examples of more subtle associations are the curl and skew or shape instability of knitted and woven goods [1].

When a fiber assembly is deformed, whether it be a woven or knitted fabric, yarn, or cord, the constituent fibers and fiber assemblies which constitute the structure are subjected to a combination of extensional, bending and torsion deformation [2]. The applied moment corresponding to unit curvature is known as bending rigidity. The bending rigidity for linear materials is the product of the tensile elastic modulus and moment of inertia of the cross-section [3].

In order to evaluate fabric hand, Pierce [4] introduced the principle of cantilever deformation in textiles to characterise fabric bending. In this method, the fabric is made to deform under its own weight as a cantilever, then the cantilever length required to produce a predetermined deflection angle is measured, and subsequently the bending rigidity of the fabric is calculated. Szablewski et. al applied numerical analysis to Pierce's cantilever beam to obtain the bending rigidity of textiles [9]. Kocik et. al used an Instron and principles of buckling in the case of small curvature to evaluate the bending rigidity of flat textiles [10].

In the case of yarn, the small dimension and untwisting of the free ends of the yarn are the major difficulties in the simple cantilever method. The exact position of the free end of a bent yarn can not be well defined and measured thus could lead to a major source of error. To avoid these difficulties in this work, a two support beam system was used to calculate the bending rigidity of yarn. The free end of the yarn is placed on a simple support, whereas the other end of the yarn is fixed on another support.

Theory

Consider an elastic beam bent by the applied moment (M). It can be shown that the curvature of the neutral line is as follows [5]:

$$\frac{1}{\rho} = \frac{M}{EI} \tag{1}$$

Where M is the moment, ρ the radius of curvature, E is the elastic modulus and I is the moment of inertia of the section. In preliminary mathematics it can be shown that the curvature of a fixed point is:

$$\frac{1}{\rho} = \frac{d^2y / dx^2}{\left(1 + \left(dy / dx\right)^2\right)^{\frac{3}{2}}} \tag{2}$$

If the length of the beam in comparison to the deflection is very large, the slope of the tangent to the curve at any point (dy/dx) is very small. In this case the value of the square of dy/dx can be neglected in compression/comparison to the unity in equation (2), and the deflection may be accepted a small deflection. The small deflection equation governing the deformation of the beam is as follows:

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI} \tag{3}$$

Figure 1 shows an elastic beam fixed at one end and supported by a simple support at the other; the beam is loaded under its own uniform weight. Using the theories for statically indeterminate beams, one can obtain [6, 7]:

$$EI \cdot y = \frac{W}{48\,l}\left(3lx^3 - 2x^4 - l^3x\right) \tag{4}$$

Where W is the total weight of the beam, and l is the length of the beam. If an external force P is applied to the beam at point B (as shown in ***Figure 2***), neglect-

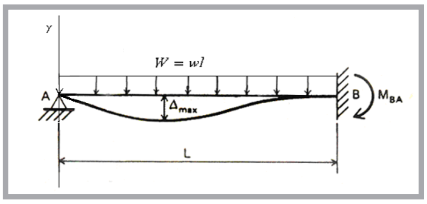

Figure 1. *Bending of elastic beam under uniform weight.*

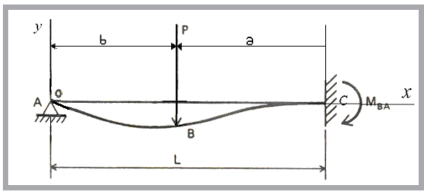

Figure 2. *Bending of elastic beam under point load.*

ing the weight of the beam, the displacement of the beam can be obtained by:

$$EI \cdot y = \frac{1}{6}\left[R(x^3 - 3l^2x) + 3Pa^2x\right]$$

From A to B (5)

$$EI \cdot y = \frac{1}{6}\left\{R(x^3 - 3l^2x) + P[3a^2x - (x - b)^3]\right\}$$

From B to C (6)

Where R is the reaction force at point A and is equal to [8]:

$$R = \frac{1}{2}P\left(\frac{3a^2l - a^3}{l^3}\right) \quad (7)$$

If there is a uniformly distributed load and point load (***Figure 2***) applied together, elastic curves (deformed shape) at different points are obtained using the superposition principle. Hence we have ***Equations 8*** and ***9***.

Two successive differentiations from the above equations lead to the coordinate of the maximum deflection point (***Equations 10*** and ***11***).

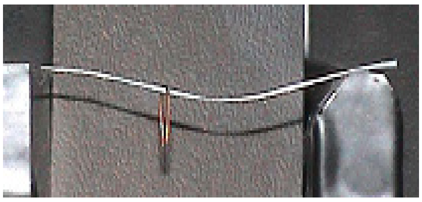

Figure 3. *The yarn on the supports. Distance between supports 30 mm.*

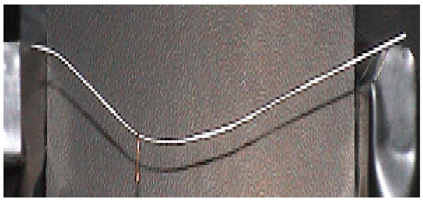

Figure 4. *The yarn on the supports. Distance between supports 60 mm.*

$$y = \frac{W}{48EIl}\left(3lx^3 - 2x^4 - l^3x\right) + \frac{1}{6EI}\left[R(x^3 - 3l^2x) + 3Pa^2x\right]$$ From A to B (8)

$$y = \frac{W}{48EIl}\left(3lx^3 - 2x^4 - l^3x\right) + \frac{1}{6EI}\left[R(x^3 - 3l^2x) + P[3a^2x - (x - b)^3\right]$$ From B to C (9)

$$\left(\frac{-8W}{48l}\right)x^3 + \left(\frac{9W}{48} + \frac{3R}{6}\right)x^2 + \left(\frac{-Wl^2}{48} - \frac{3Rl^2}{6} + \frac{3Pa^2}{6}\right) = 0$$ From A to B (10)

$$\left(\frac{-8W}{48l}\right)x^3 + \left(\frac{9W}{48} + \frac{3R}{6} - \frac{3P}{6}\right)x^2 + \left(\frac{3Pb}{6}\right)x + \left(\frac{-Wl^2}{48} - \frac{3Rl^2}{6} + \frac{3Pa^2}{6} - \frac{3Pb^2}{6}\right) = 0$$ From B to C (11)

Equations: 8, 9, 10, and 11.

Experimental

A type of zero twist PET filament yarn was used in this experiment. The yarn count was 150 dtex consisting of 48 filaments. In order to have two supports, a small vice was used. Two moveable jaws of the vice were used as supports. The distance between the jaws, i.e. the distance between the supports, could be adjusted with the handle of the vice.

A special cellar tape was fixed to the support to act as a fixed end. The other support acted as a simple support. A very light weight made of thin copper wire was softly placed on the yarn near the fixed support . The weight of the wire was measured to the nearest 0.1 of a milligram and found to be 0.0041 grams.

The experiments were carried out on 20 samples of the yarn, and for each sample seven different distances between the supports were tested. For each sample the distance between the two supports was chosen as 30, 35, 40, 45, 50, 55 and 60 millimeters, and the length of the yarn was 10% more than the distance between the two supports. ***Figures 3*** and ***4*** show two typical photographs of the bent yarn.

In order to increase the accuracy of the experiments, photographs of the yarn in each experiment were taken using a digital camera, and then by exerting a suitable resolution on each photograph, the deflection of the yarn, coordinates of the point of applying the external load and the point of maximum deflection were obtained.

In order to confirm the validity of the small deflection equations, the coordinates of the various points of the bent yarn were obtained and compared with the shape of a elastic beam curve with the same characteristics in small deflection . If the curves are close to each other, we can use small deflection equations to obtain the bending rigidity of the yarn; otherwise the large deflection case must be considered.

This investigation was conductedusing three samples of the yarn. Each sample contained seven different distances between the supports. For each experiment, graphs of the shape of the yarn deflection and the shape of the beam deflection curve, in the same condition, were prepared. For all the samples investigated, it was observed that, in the small lengths of the beam, the curve of the yarn deflection was close to that of the elastic beam deflection. More deviation was observed as the length of the beam increased. ***Figure 5*** shows one of the graphs indicating the deflection of the yarn and elastic beam in the same experiment.

Subsequently, by illustrating that we can use elastic beam equations in cases of

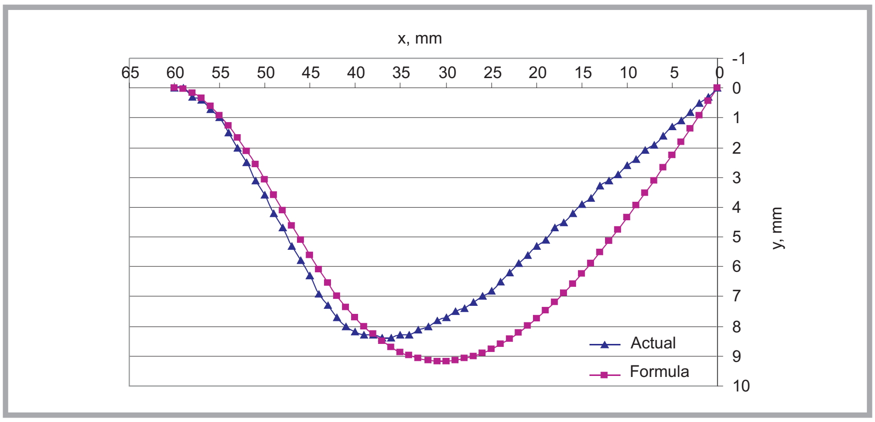

Figure 5. *Curves of deflection of yarn and beam when distance between two supports is 60 millimeter.*

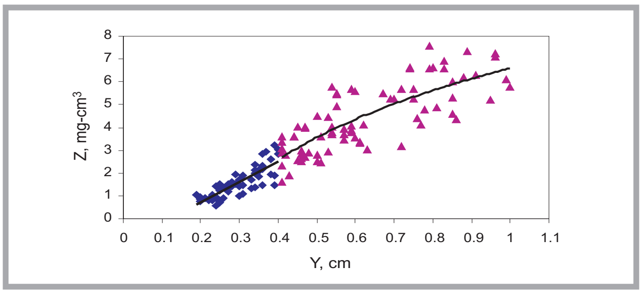

Figure 6. *Plots of Z versus Y.*

small deflection, and by preparing a suitable computer program, the bending rigidity of the yarn was calculated.

Results

Determination of yarn bending rigidity

As was mentioned before, in the case of small deflection, ***Equations 8*** and ***9*** can be used to calculate the bending rigidity of yarn. These equations can be written as $Y = Z/EI$, where Y is the coordinate of the maximum deflection point. If the small deflection equations hold, plots of the values of Z versus Y must be a straight line with a slope equal to EI. The values of Z for each sample were obtained by using ***Equations 8*** and ***9***. ***Figure 6*** shows the plots of Z values obtained for the corresponding Y for all experiments.

As can be seen from ***Figure 6***, the data points are close to each other in the small beam lengths, and more variations are observed as the length of the beam increases. It is also clear that two different parts can be distinguished. At small values of deflection, $Y \leq 0.4$ cm, the data points show a linear trend confirming the linearity of the data points and validity of small deflection equations. However, as the deflection increases, $Y > 0.4$ cm, the data points deviate from the straight line, indicating the onset of large deflection and less accuracy in cases of small deflection. The reason for this is that the accuracy of small deflection assumptions decreases as the length of the beam increases.

The bending rigidity of the yarn was calculated from the slope of the first part of the curve and found to be $B_y = 8.84 \times 10^{-6}$ Kg·cm^2, which is equal to 8.84 mg·cm^2. The linear correlation coefficient of the data points in the first part was also found to be $R = 0.842$.

Conclusion

Yarn was considered as an elastic beam deflected by applying an external point load. Simulation was performed by settling one end of the yarn on a fixed support and the other on a simple support. The results showed that the behaviour of the yarn tested in this condition follows from the behaviour of the in the small deflection case. Therefore, in order to calculate the yarn bending rigidity, small deflection equations can be used with acceptable accuracy.

References

1. Platt M. M., Klein W. G., Hamburger W. J.; Mechanics of Elastic Performance of Textile Materials, part XIV: Some Aspects of Bending Rigidity of Singles Yarns, Textile Res. J. **29**, 1958, pp. 611-627.
2. Zorowski C. F., Chen C. S.; Cantilever Bending Behavior of Continuous Filament Cords, Textile Res. J. **35**, 1965, pp. 529-538.
3. Cooper D. N. E.; The Stiffness of Woven Textiles, J. Textile .Inst. **51**, 1960, pp. 317-335.
4. Pierce F. T.; The Geometry of Cloth Structure, J. Textile Inst. **28**, 1937, pp. 45-97.
5. Timoshenko S., Young, D. H.; "Element of Strength of Materials", Published by Van Nostrand Company, New York, 5th Edition 1969.
6. Gere J. M., Timoshenko S. P.; "Mechanics of Materials", Second SI Edition, PWS Publishers, 1984.
7. Oberg E., Jones F. D.; "Machinery Handbook", Industrial Press Inc., New York, 19th Edition 1971.
8. Roark R.J., "Formulas for Stress and Strain", McGraw-Hill Book Company, 1965.
9. Szablewski P., Kobza W.; Numerical Analysis of Pierce's Cantilever Test for the Bending Rigidity of Textiles, FIBERS & TEXTIELS in Eastern Europe, October/December 2003, Vol. 11, No. 4(43).
10. Kocik M., Zurek W., Krucinska I., Gersak J., Jakubczyk J.; Evaluating the Bending Rigidity of Flat Textiles with the Use of an Instron Tensile Tester, FIBERS & TEXTIELS in Eastern Europe, April/June 2005, Vol. 13 No. 2(50).

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