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A Discrete Probalistic Model of Forces in a Visco-elastic Thread Pulled Through a Drawing Zone

Abstract
A simulation was carried out of forces acting in a thread of random variable visco-elastic rheological properties, which is displaced through a model drawing zone. This zone consists of two pairs of rollers working at different tangential velocities. The computer simulations carried out indicated a significant influence of irregularities in the thread's visco-elastic properties on the stochastic character of the thread tensions. The latter decrease with an increase in the length of the drawing zone, and rise with the velocity of thread displacement. The model described in this article is a development of the model based only on the elasticity theory elaborated previously by the autors, and also described in Fibres & Textiles in Eastern Europe.

Key words: *visco-elastic thread, random irregularities, tensions, probalistic model, drawing zone.*

Designations

F_0, in cN – preliminary tread tension before the drawing zone,

F, in cN – thread tension in the drawing zone,

C, C_1, in cN – relative coefficients of tensile rigidity for the Zener model (Figure 1); the average values of these coefficients are accepted as input data in the calculation algorithm,

C_1, C_2, ... , C_n; C_{1n}, in cN - relative coefficients of tensile rigidity C, in cN, of the subsequent thread segments, for the branch of the Zener model which represents the elasticity after random modification,

C_{11}, C_{12}, ... C_{1n} - relative coefficients of tensile rigidity C_1, in cN, of the subsequent thread segments, for the branch of the Zener model which represents the visco-elasticity after random modification,

nn – coefficient of variation of the relative coefficients of the tensile rigidity C and C_1 in the Zener model, accepted for calculation,

η, in cNs – relative dynamic viscosity of the thread matter,

ε - relative elongation of the thread forced in the drawing zone,

t, in s – time,

$v_\varepsilon = d\varepsilon/dt$ – speed of the deformation increase in the thread deformation,

ΔL, in m – absolute elongation of the thread segment in the drawing zone,

V_1, in m/s – output speed of the thread (out of the drawing zone),

V_0, in m/s – input speed of the thread (feeding speed into the drawing zone),

L_S, in m – length of the zone,

L_0, in m – length of the thread segment,

C_ε, in cN – equivalent relative coefficient of the tensile rigidity for n segments in the drawing zone determined for the dependency $F = C_\varepsilon \varepsilon$ [1],

$C_{\Delta L\ EQUIV}$ - equivalent relative coefficient of the tensile rigidity for n segments in the drawing zone determined for the dependency $F = C_{\Delta L}\Delta L$ [1],

k – number of the thread segments which will be displaced through the drawing zone in the algorithm.

Introduction

Investigations into the dynamic properties of threads and fibres carried out indicate that yarn should be considered as a viscid-elastic body. This statement concerns yarns of natural fibres, for example cotton, as well as those of synthetic fibres. In textile technologies, such as spinning, weaving, knitting, and texturing, different drawing zones of threads can be found. These zones are positioned between machine-elements and on friction barriers through which threads are pulled. The force occurring in threads is the basic parameter of all the manufacturing processes of textile products. The efficiency of machines and the final product quality depend on the value of this force and its characteristic changes. The variability of forces in threads subjected to textile processes is caused by technology forcing and factors connected with irregularities in the mechanical properties of threads. Hitherto, while modelling the forces in threads, the variability aspect of their values caused by their mechanical properties is usually omitted. Therefore, the modelling results do not determine the parameters of force dispersion, which occur in real processes.

This was the reason that the authors made an attempt to develop, on the basis of works earlier described [1, 2], a model of forces in a thread displaced through a drawing zone, which considers the randomly variable values of the visco-elastic thread parameters differentiated along the thread's length [3 - 11, 12, 13, 16 - 21].

Experimental research [22, 23 - 26] testify that there a significant influence of the thread pulling-through velocity and at the same time the speed of the increase in relative thread deformation on the value of forces. It can be observed that higher deformation speeds result in higher tensions than those created when slowly drawing. This phenomenon is essential, as it can significantly increase the tensions in high-velocity processes, characterised by the high drawing velocities of the threads pulled-through.

Therefore, in our further investigations on the basis of an improved model, we made the attempt to present the influence of random changes in the material's rheological visco-elastic properties in short segments on the generation and characteristic of forces in a thread transported through a drawing zone. Simplified considerations based on the model, which concerns only the theory of elasticity we presented in [1]. The aim of this work was to indicate significant differences in the characteristics of thread tensions when those assumptions were accepted, which are more realistic and consider the visco-elastic properties of a thread.

Physical basis of the considerations

The behaviour of threads under the conditions of the processes of dynamic drawing as well as the relaxation and creep of forces can be presented with the use of the rheological three-element Zener model (Figure 1) [27].

The Zener model is composed of two parallel branches. The first includes only a spring and represents the elastic properties which cause deformations directly proportional to the force. These features are characterised by the relative coefficient of tensile rigidity C, in cN. The second branch represents the visco-elastic properties. It is composed of two elements connected in series: a spring whose properties are characterised by the relative coefficient of tensile rigidity C_1, in cN, and a damper whose ability to deform under the influence of a tensile force is characterised by the viscosity coefficient η, in cNs.

The dependence between the deformation ε, the tensile force F, the time t of the force action, and the rheological parameters of the Zener model C, C_1, and η are described by the following differential equation:

$$F + \frac{\eta}{C_1} \cdot \frac{dF}{dt} = C \cdot \varepsilon + (C + C_1) \cdot \frac{\eta}{C_1} \cdot \frac{d\varepsilon}{dt} \quad (1)$$

In the case during the deformation speed

$$\frac{d\varepsilon}{dt} = const = v_{\varepsilon} \text{ and the relative deformation } \varepsilon = \frac{d\varepsilon}{dt} \cdot t = v_{\varepsilon} \cdot t \quad (2)$$

(tension change at constant deformation speed), the solution of equation (1) is given by the dependency:

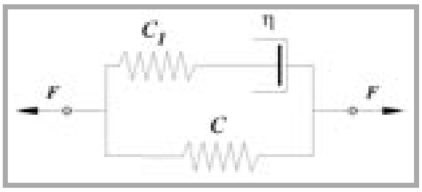

__Figure 1.__ Three-element Zener model, C, C_1 – relative coefficients of tensile rigidity, η - viscosity.

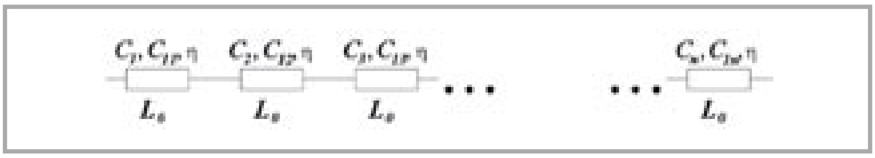

__Figure 2.__ The assumed rheological model of a thread.

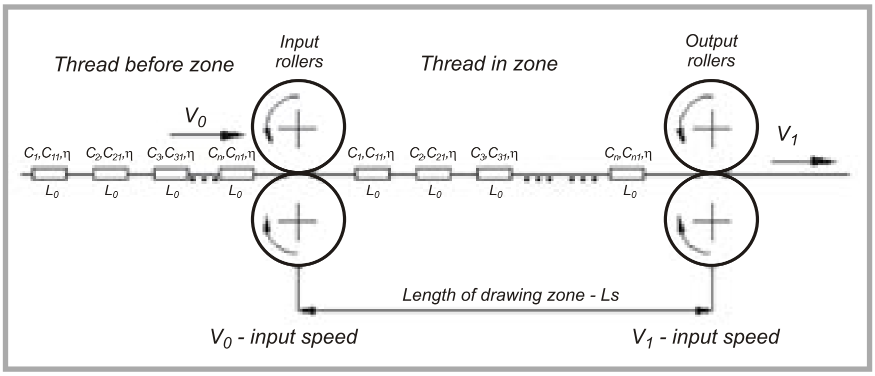

__Figure 3.__ A model of the drawing zone; the model is similar to that in [1], but each subsequent thread segment (element) is a Zener model, whereas in [1] each segment was a Hook model.

$$F = F_0 \cdot e^{\frac{-t \cdot C_1}{\eta}} + C \cdot \varepsilon + \eta \cdot v_{\varepsilon} \cdot \left(1 - e^{\frac{-t \cdot C_1}{\eta}}\right) \quad (3)$$

Equation (3) describes the tension changes of a thread while it is stretched with a constant velocity increase in relative deformation. In calculations hitherto carried out, as for example in [9], the values of the coefficients C, C_1, and η, which are material constants, is assumed to be constant along the whole length of the thread transported through the drawing zone. Therefore, the tension values obtained at this assumption from the dependencies described above are assumed to be average, which do not give any information about the variability of tensions and their dispersion, which is indicated by performing experiments.

Assumptions and theoretical basis of our considerations

The thread model shown in Figure 2, and the structure of a drawing zone presented in Figure 3 were assumed in order to model the variability of forces acting in threads while considering their rheological properties.

Assumptions

- The thread is composed of short segments (elements). Each of these segments is a Zener model with properties defined by the coefficients C_n (in cN): C_1, C_2, … , C_n; C_{1n} (in cN): C_{11}, C_{12}, … C_{1n}; and the viscosity η (in cNs).
- The values of C and C_1 of the particular segments change randomly and are characterised by a normal distribution.
- The dependence between the relative elongation (deformation) ε, the tensile (drawing) force F and the drawing time t is determined by the dependency (3).
- The relative elongation ε assumed is accepted as constant in the drawing zone for the whole drawing process (ε = constant).
- The conditions determined by dependency (2) are maintained, which means that the velocity of the increase in segment deformation (the segment which entered into the drawing zone) is constant.
- The time of drawing the thread segment, which is placed in the drawing zone, is equal to the time of displacement through the drawing zone with the input (feeding) speed.
- Equivalent values of the coefficients C and C_1 of the stretched thread length composed of n elements, which are actually present in the drawing zone, are determined by the following dependencies: $C_\varepsilon = C_{\Delta L} \cdot L_0$ and $\frac{1}{C_{\Delta L\Sigma}} = \frac{1}{C_{\Delta L1}} + \frac{1}{C_{\Delta L2}} + \frac{1}{C_{\Delta L3}} + \dots + \frac{1}{C_{\Delta Ln}} = \sum_{i=1}^{n} \frac{1}{C_{\Delta Li}}$ [1, 2].
- The simplified displacement of the thread through the drawing zone, which means the subsequent calculation of the force, is carried out after exchanging extreme elements of the thread segment in the zone.

- Before performing the calculations, the following parameters should be assumed:
  - the average values (in cN) and C_1 (in cN) together with their coefficients of variation,
  - viscosity η (in cNs),
  - length of the elementary thread segment L_0 (in mm),
  - length of the drawing zone L_s (in mm),
  - input (feeding) speed V_1 (in m/s),
  - relative elongation $\varepsilon = (V_1 - V_0)/V_0$.

Calculation algorithm

The random modification of the average values *C* and C_1 undertaken by the program is based on processing by a computer, considering the average value and the accepted by us coefficient of variation. The coefficients C and C_1 are random modified, mutually independently. The calculation algorithm is presented in Figure 4.

Analysis of the model

As an example, Figures 5.A and 5.B present the histograms of sets of random numbers used for our calculations. Values of the force which stretches the thread positioned in the drawing zone are obtained as the result of calculation; the runs of the forces in dependence on the length of the already transported thread segment, for different lengths of the zone, are presented in Figure 6. The length values of the zone are marked on the drawing. It is visible that the tensions obtained have a variable stochastic character. Figures 7.A and 7.B show examples of histograms of the values obtained by calculation.

On the basis of an analysis of the dependency results (3), we can state that the algorithm developed by us for calculating the instantaneous tension values of the thread in the drawing zone considers the influence of the transporting speed of the thread through the zone on the tension values obtained.

The 3-D dependency presented in Figure 8 (see page 48) illustrates changes in the average tension in the zone with dependence on the zone's length and the accepted coefficients of variation of *C* and C_1. From this dependency it results that according to the model developed, the average tension in the drawing zone decreases with an increase in the zone's

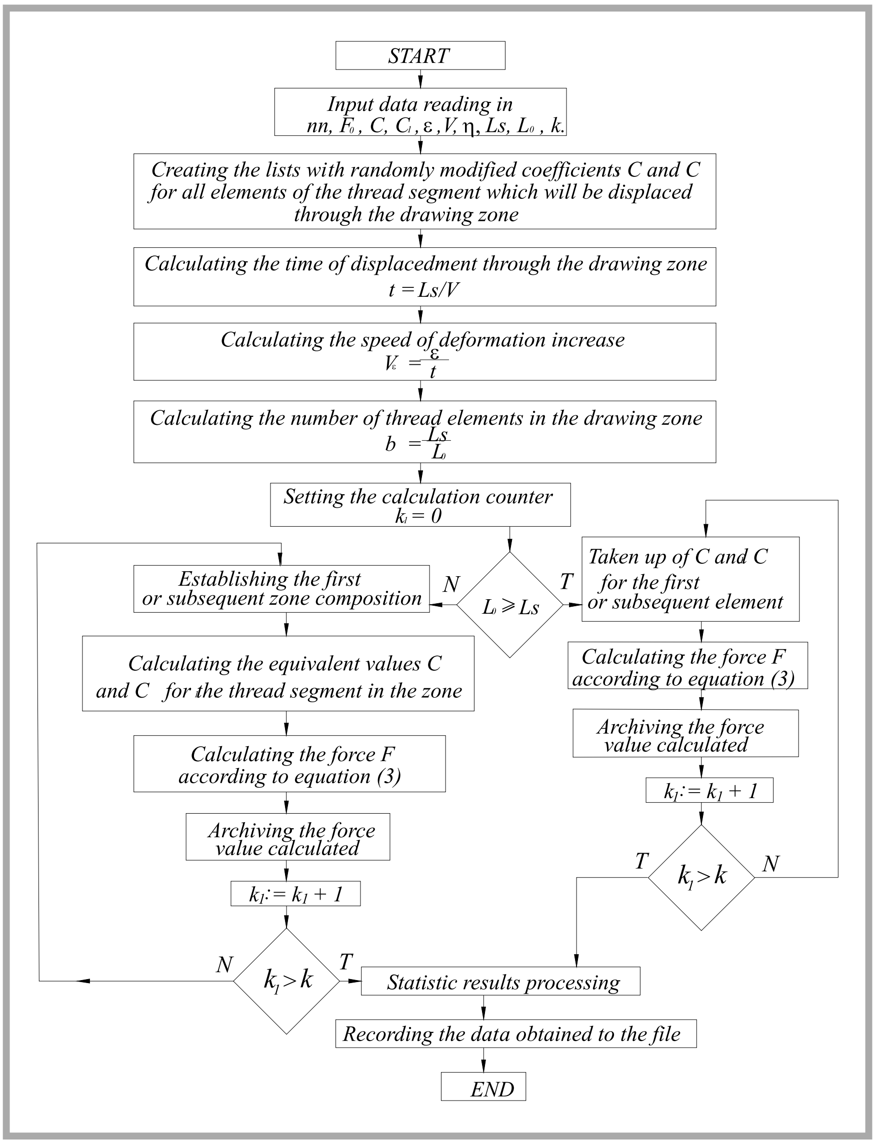

Figure 4. Scheme of the algorithm for calculation of the forces in a thread transported through a drawing zone which considers the random visco-elastic properties of yarn (thread).

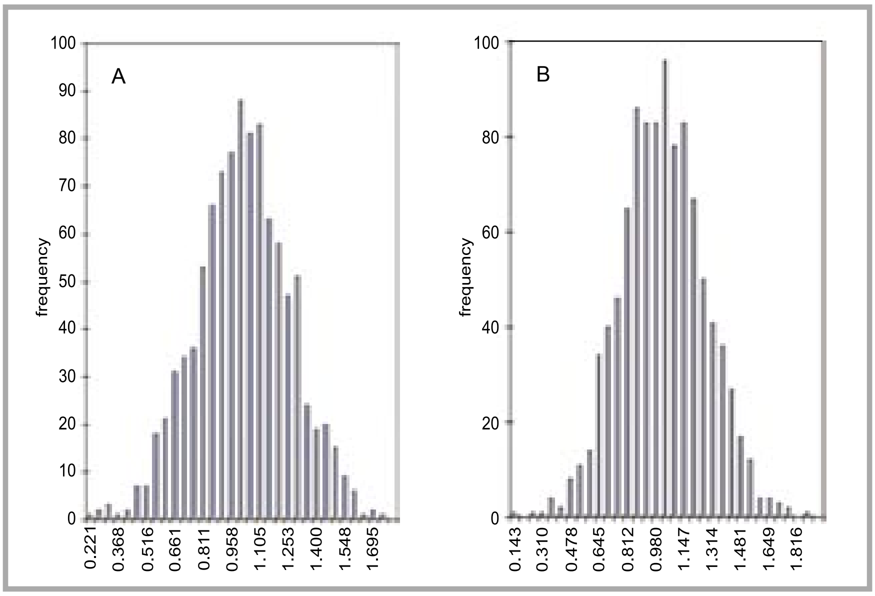

Figure 5. Histogram of random numbers used for modifying the coefficients of tensile rigidity C and C_1: A – for coefficient C; B – for coefficient C_1.

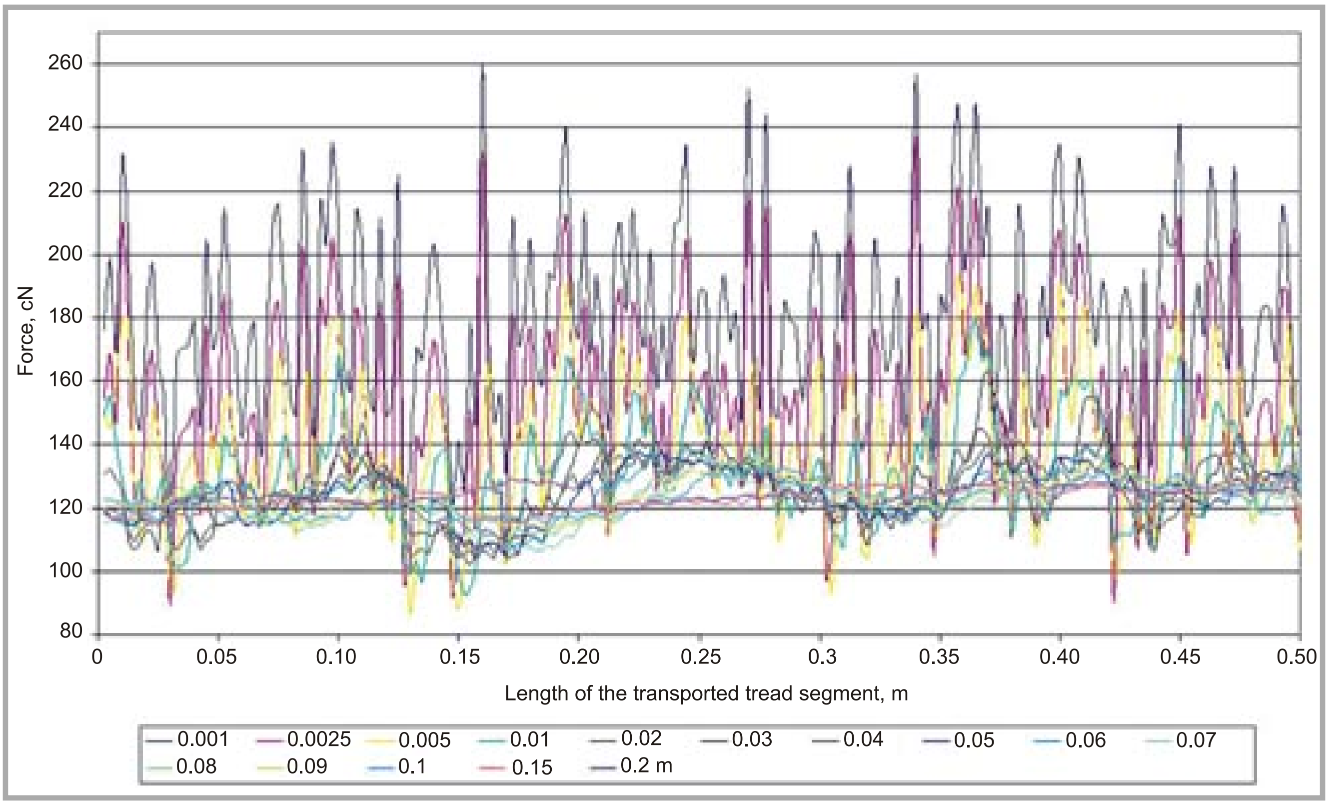

Figure 6. Variable thread tensions in the drawing zone obtained by the algorithm shown in Figure 4 as a function of the length of the already transported thread segment. Particular runs obtained from the model for different lengths of the drawing zone are listed at the bottom of the figure. ***Note:*** *All dependencies are drawn in colour only in the internet-edition of the journal. Runs with smaller tension dispersions were obtained for longer drawing zones (three characteristic waves of longer wavelength). Runs with a higher average value and higher amplitude were obtained for small drawing zone lengths.*

length and an increase in the coefficients of variation of C and C_1 . On the other hand, the dependency presented in Figure 9 indicates that the rheological model developed considers the influence of the thread's speed on the average tension in the drawing zone, which increases with an increase in the displacement speed of the thread anda decrease on the zone's length. This phenomenon reveals, therefore, that the deformation speed the thread is higher for shorter drawing zones, at the same transporting speed of the thread through the zone. It should be emphasised that this means that the character of the results obtained by the model is in conformity with experimental observations. The influence of the transporting speed of the thread is a factor which hitherto has not been considered while analysing the model described in [1].

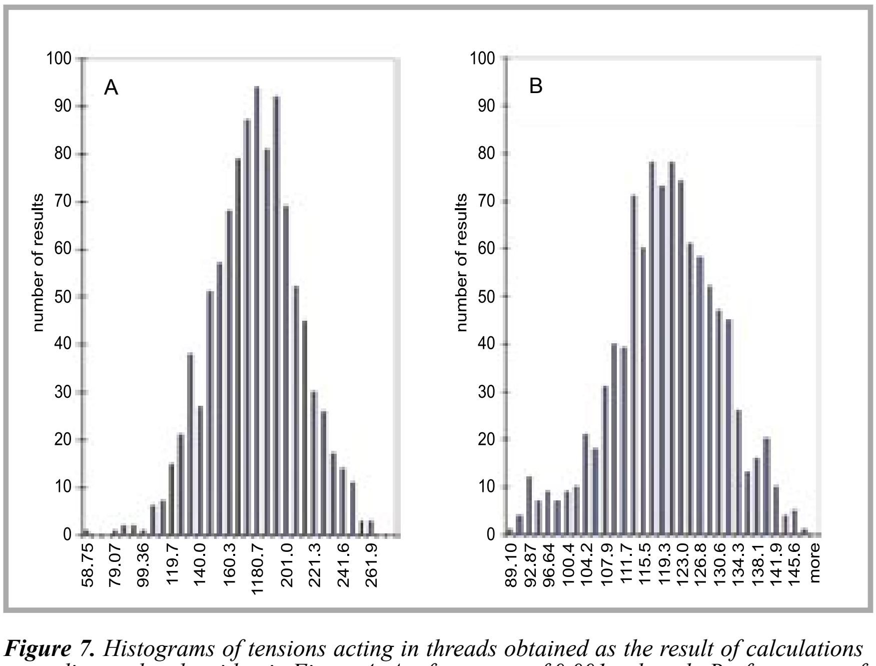

Figure 7. Histograms of tensions acting in threads obtained as the result of calculations according to the algorithm in Figure 4; A – for a zone of 0.001 m length, B - for a zone of 0.03 m length.

The considerations presented above confirm that the forces in threads increase as a result of the phenomenon that the relaxation process of the forces in threads does not follow an increase in their values. This can be explained by the interpretation of the formula exp(-t C_1/η), which is responsible for the run of the relaxation process, and indicates that the forces depend inversely and exponentially on the time of deformation. This time depends on the transporting speed and the length of the zone. As is visible from the 3-D dependencies shown in Figures 8 and 9 (see page 48), the variability of the rheological properties along the thread only lower the average tension value, although it does not influence the remaining relations determined by the dependency (3).

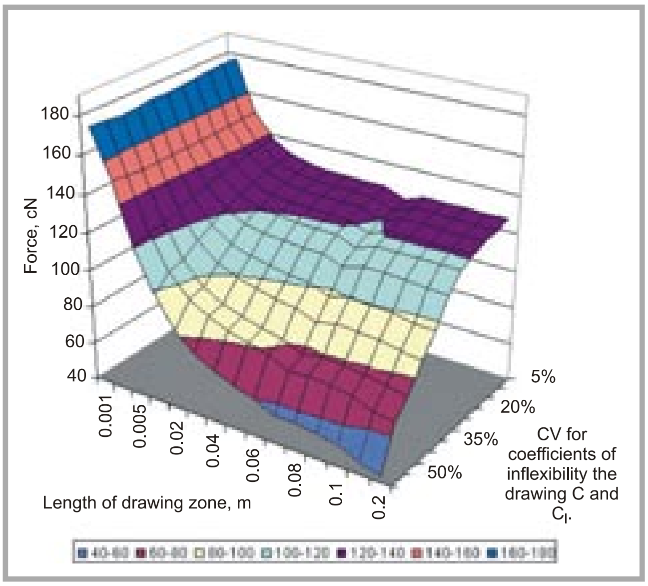

Figure 8. Average value of tension with dependence on the length of the drawing zone and the coefficient of variation of the tensile rigidities C and C_1.

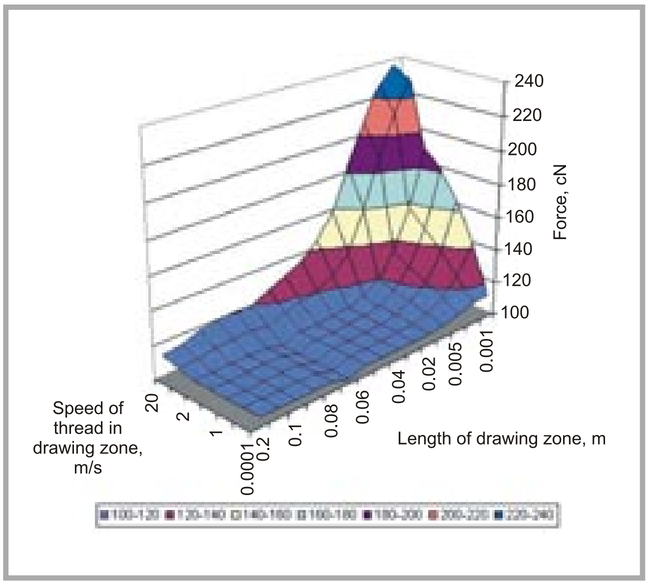

Figure 9. Influence of the thread transporting speed through the drawing zone and the length of the zone on the average thread tension in the drawing zone.

On the other hand, the dependency presented in Figure 10 indicates the formation of the values of the coefficient of variation of the average value of the thread tension in the drawing zone obtained by the model. It is visible that the values of the coefficient of variation of the tension are greater for short drawing zones. The curves presented in this Figure concern calculations carried out with different accepted values of the coefficients of variation for the tensile rigidity C and C_1. While comparing the values of the coefficient of variation obtained for the forces acting in the zone with the accepted values of coefficients C and C_1, we can state that the value of the coefficient of variation of the force in the drawing zone is lower than the values of the coefficients of variation of the parameters C and C_1 accepted for calculation.

The value of the coefficient of variation of the tension in the drawing zone decreases with an increase in the length of the zone, and is higher if the elementary thread segments are longer. This phenomenon is characteristic for the model described in [1], as well as for that discussed in this article.

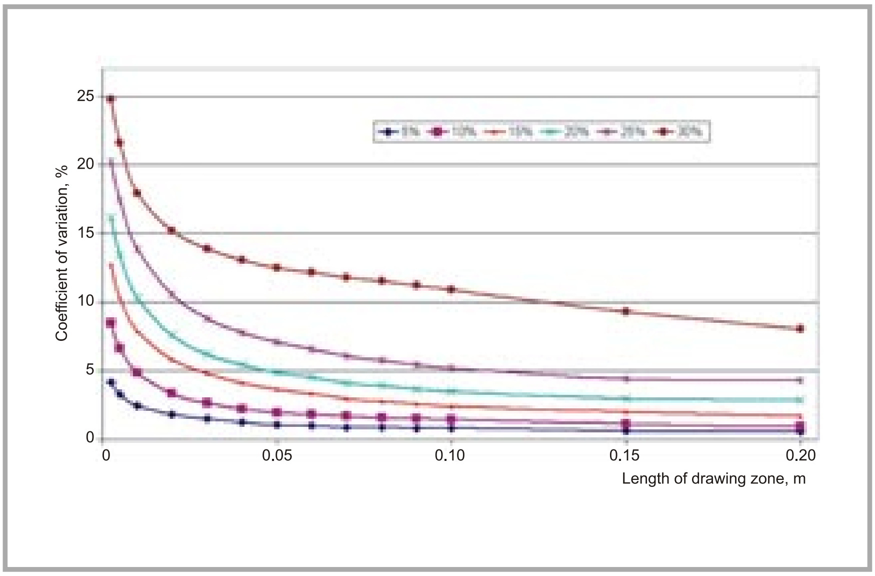

Figure 10. Influence of the length of the drawing zone on the coefficient of variation of the thread tension in the drawing zone for different accepted coefficients of variation of the tensile rigidities C and C_1.

The value of the coefficient of variation of the thread tension obtained by the model also depends on the assumed value of the coefficient of variation. This dependency can be described as the following: the coefficient of variation of thread tension decreases with an increase in the length of the drawing zone, but its value is always lower than the assumed value of the coefficients of variation accepted for the calculation, and the average values of C and C_1. This dependency is illustrated by the graphs in Figure 10. It should be stressed that this dependency occurs irrespective of the assumed values of the coefficients C and C_1, as the assumed average value of C and C_1 influences only the average value of the thread tension.

An experimental verification of the results of the theoretical investigations described above was carried out with the use of a special thread (yarn) tester with variable length of the drawing zone which we designed and constructed [2]. The tests performed confirmed the correctness of our theoretical considerations.

Conclusions

- The simplified model of displacing a thread through a drawing zone, which considers the thread's rheological properties together with probabilistic elements, generates instantaneous values of the thread tension, which depends on the length of the zone and the speed of drawing.

Może rys 10 ma Pan też w Excelu?

- The model elaborated considers the influence of the zone's length and the deformation speed of the thread on the tension value and its characteristics.
- The variability of the thread tension rises with the shortening of zone's length.

Acknowledgment

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