Agnieszka Serwatka,
***Pascal Bruniaux,**
Iwona Frydrych

Technical University of Lodz
Faculty of Textile Engineering and Marketing
Department of Clothing Science and Technology
ul. Żwirki 36, 90-942 Łódź, Poland

***Laboratoire GEnie et Matériaux TEXtiles (GEMTEX), UPRES EA2161,**
École Nationale des Arts et Industries Textiles (ENSAIT),
9 rue de l'Ermitage, BP 30329, 59056 ROUBAIX Cedex 01, France

New Approach to Modeling the Stress-Strain Curve of Linear Textile Products. Part 1 – Theoretical Considerations

Abstract
The first part of this paper presents a modelling of the stretching phenomena of linear textile products, specifically yarns. The stretching phenomena are described by a model of the yarn stress-strain curve which was developed by a team from the GEMTEX laboratory. This analysis of the yarn stress-strain curve is an attempt to interpret this curve in the form of a set of various models. This differentiation is due to the change of the yarn's state at the time of its stretching. The model differs very clearly from the other models presented in literature. In order to compare our research with that described in literature, the sensitivity of the parameters of various models was studied. Dividing the model into sub-models is crucial, because this enables the identification of vector parameter as a whole, by assembling parameters of sub-models. The second part of this approach to modelling the stress-strain curve is concerned with modelling the stress-strain curves of real natural and synthetic yarns.

Key words: *stretching, stress-strain curve, simulation, modeling, yarn, sensitivity.*

List of symbols

T - stretching force,
ε - elongation,
E - Young's modulus,
$\zeta, a, d, c, A, B, C, D, E, a_1, b_1, c_1, d_1, e_1, f_1$ - parameters of the models,
$T^*, T^{**}, \varepsilon^*, \varepsilon^{**}$ - distinct points for 3 zones of the stress-strain curve,
r - velocity of the elongation increase,
t - time,
η - Newton's modulus,
$T_r = T_0$ - fibre's pre-tension,
b - coefficient of nonlinearity,
P - friction effect,
M - inertia effect.

Introduction

Textile engineers have known for a long time that the properties of fabrics have an essential influence on the manufacturing process, as well as on the appearance of ready-to-wear clothing. On the basis of established knowledge about textiles, they tried to predict the garment behaviour. Although the earliest known investigations were carried out in the second half of the nineteenth century (Tchebichef [1] and Lucas [2]), the level of knowledge remains limited. The structure of fabrics and their properties are very complex, because of the following factors:

- the variety of thread interlacing due to different weaves (plain, twill, satin),
- the variety of raw materials used, which can be natural like cotton, wool, flax and silk, or synthetic like PA, PES, PP and many others,
- the physical characteristics of the materials which compose the fabric.

Economical and industrial reasons are strong motivations for research into the models of yarn and fabrics, the objective of which is to understand the dynamic behaviour of teh latter. Nowadays, developments in computing make possible a virtual simulation of fabrics and threads. In order to characterise the properties of fabrics, it is first necessary to consider their structure. In this case, a fabric's geometry should be established with great precision on the basis that it is a result of the interlacing of warp and weft yarns. Consequently, a fabric modelling requires the modelling of the yarn.

The aim of our study was to elaborate a new model of the stretching behaviour of yarn by a new mathematical description of the stress-strain curve. The new model elaborated is compared with other models hitherto used in textile science.

Theoretical

Proposed model

On the basis of many practical tests carried out on various samples, we were able to estimate a general shape of the yarn stress-strain curve $T(N)=\zeta\varepsilon(\%)$, which is presented in Figure 1. This curve is divided into three distinct parts, which were described below.

The first zone

The first zone of the curve corresponds to the moment when the tensile tests of the yarn begin. The yarn is a fibre assembly. The fibre arrangement in the yarn is partially ordered, although during the phase of yarn design, the process of spinning tries to preserve a regular torsion, which makes it possible to maintain its homogeneity. During the tensile tests,

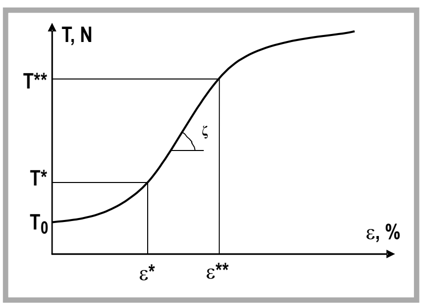

Figure 1. *Model of the stress-strain curve.*

the fibres are subjected to longitudinal acting stress. The resulting strain causes an alignment of the fibres, which generates the interaction of fibres as well as the phenomenon of slippage. This zone corresponds to the ‘pre-tensioning’ of fibres; the fibres are oriented according to the axis of deformation. Thus the fibrous structure tends towards a new space orientation (the principle of minimum energy), before the deformation of yarn starts from the point ε*.

The second zone

This zone explains the elastic character of the yarn (the zone with linear deformation); the relationship between stress and deformation is proportional within this area. The proportionality factor is represented by Young’s modulus. The value of this modulus is constant until a certain maximum limit, beyond which the deformation is no longer elastic (the plastic zone). The process of passing from the elastic zone to the plastic deformation zone is identified by the point ε**

The third zone

The last zone of the stress-strain curve reveals nonlinear phenomena, which are explained by the damage of fibres. During this phase, there is a progressive destruction of fibres, starting from the most strained ones.

Approach

The analysis of the stress-strain curves allows a model of the curve to be established. In order to take a systematic approach to the problem, we observed the stress-strain curve like a signal in time, i.e. the stress was compared with the dynamic response of a system; the deformation corresponds to the time. In the first phase, we analysed the models established by other researchers. We studied (for each model) the sensitivity of each single parameter, i.e. the degree at which a selected single parameter influences the shape of the curve, while other parameters remain constant. This approach allows us to estimate the influence of each parameter in time and checks, if our initial hypothesis of dividing a time space into three distinct zones is feasible. We must emphasise that this division of our model introduces a discontinuity in the boundary points of each zone. This study leads us to different sub-models acting in each zone of our model. To facilitate the comparison of each model with our model, a simulation program was written using MATLAB software. The aim of the development is to show the meaning of each variable, i.e., its influence on the shape curve.

$$
\begin{cases}
T = \zeta\left[\left(1-\dfrac{T^{*}}{\zeta\varepsilon^{*}}\right)\varepsilon + \dfrac{2T^{*}}{\zeta} - \varepsilon^{*}\right]\left(\dfrac{\varepsilon}{\varepsilon^{*}}\right) + T_{0} & \varepsilon \le \varepsilon^{*} \quad (1)\\
T = \zeta\left(\varepsilon - \varepsilon^{*}\right) + T^{*} & \varepsilon^{*} \le \varepsilon \le \varepsilon^{**} \quad (2)\\
T = \zeta\left[1-\left(a + d\,\exp\left(-\dfrac{\varepsilon}{c}\right)\right)\right]\left(\varepsilon - \varepsilon^{**}\right) + T^{**} & \varepsilon \ge \varepsilon^{**} \quad (3)
\end{cases}
$$

Equations 1-3.

The shape of the curve is described by the Equations (1–3) where: T expresses the tension, ε presents the strain, and the coefficients ζ, *a, d, c* are the parameters of the model.

Sensitivity of the proposed model

Our model combines the various dynamic components present in the other models. Only a study on the sensitivity of parameters of each model enables to show them [8]. In order to show the influence of each parameter on the models, it was necessary to change their values in a particular area (± 5% of initial value) and simultaneously to maintain the other parameters unchanged, as illustrated in Figure 2. We will thus demonstrate that the parameters of our model contribute to a good representation of the curve by acting similarly to the parameters of the other models further discussed.

The particular parameters have the following meaning:

- The parameter ζ is a variable, which influences the slope of the curve in the three zones identically. It ensures that the function continuity is maintained in the discontinuity points of the sub-models, as the tangents at the interface boundary points must be the same,
- The role of parameters T^{*} and T^{**} is to avoid the break at points of discontinuity,
- The parameters a, d, c only perform in the third part of the curve. This part has a nonlinear character, and is man-

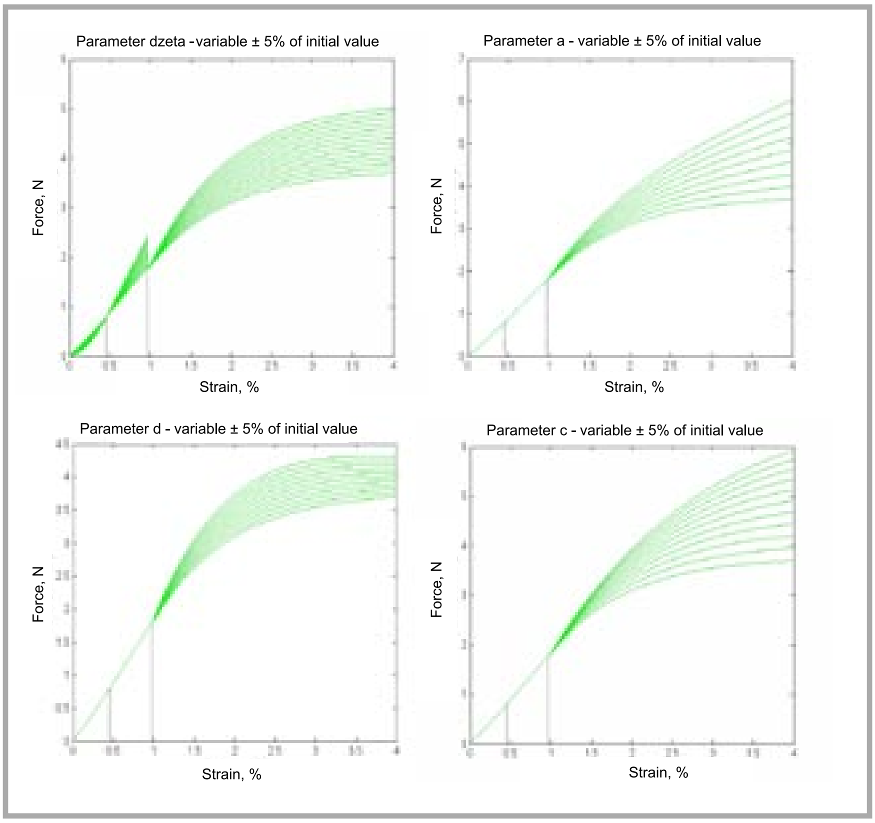

Figure 2. *Curves of sensitivity of the model proposed.*

aged in the following way, as shown in Figure 2. More exactly, the parameter d as a time constant influences the behaviour of the signal, balances the final level of the curve and is equal to the variable factor of intensification. Parameter a tends to give a non-zero slope of the signal, whereas parameter c amortizes this effect, especially at the end of the signal.

Models of traction (tensile models of the stress-strain curve)

The following models are discussed as comparison for the model proposed by the authors: the Vangheluwe's model, the Żurek's model, the Vangheluwe's model modified by Manich, the Żurek's model modified by Manich , and the Legrand's model.

Vangheluwe's model

Vangheluwe's model [3, 4] is presented in Figure 3.

This model is a combination of various models. It consists of Hooke's model characterised by E (elastic model) connected in series with a visco-elastic model characterised by η - Maxwell's model, and in parallel with set $T_1 = b\varepsilon^2$ in order to take into account the nonlinearity caused by the stretching forces.

The equation for this model is:

$$T(\varepsilon) = T_r + \eta r[1-exp(-E\varepsilon/\eta)] + b\varepsilon^2 \quad (4)$$

where T_r means pre-tension of fibres,

By the following designations:

$$\eta r = A \quad (5)$$

$$-E/\eta = -B \quad (6)$$

$$b = C \quad (7)$$

$$T_r = T_0 \quad (8)$$

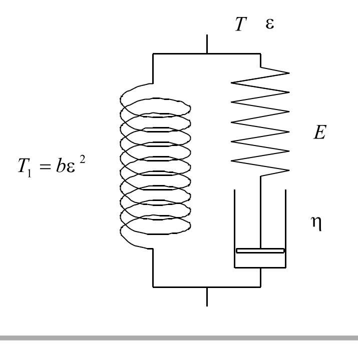

***Figure 3.** Vangheluwe's model.*

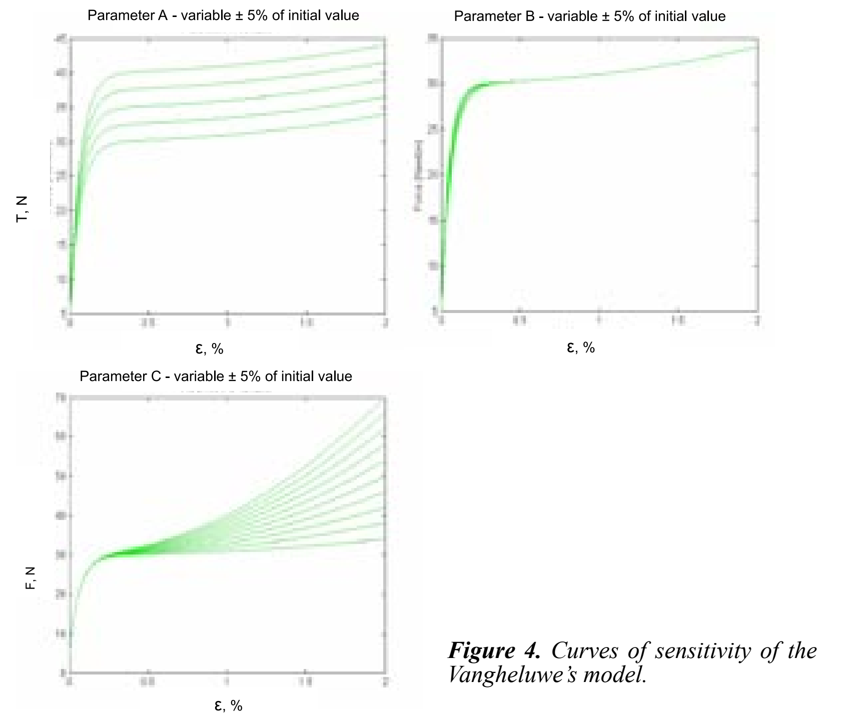

***Figure 4.** Curves of sensitivity of the Vangheluwe's model.*

and introducing the parameters *A, B, C*, and T_0 in equation (4), the final equation is as follows:

$$T(\varepsilon) = T_0 + A[1-exp(-B\varepsilon)] + C\varepsilon^2 \quad (4)$$

Sensitivity of the Vangheluwe's model

The sensitivicties of the parameters used in equations characterising this model are presented in Figure 4. In Vangheluwe's equation, an expression which is very close to Maxwell's model supplemented by a nonlinear spring is introduced. This factor describes the initial state of the pre-tensioning of fibres in the yarn. From the analysis of curve sensitivity, we could draw the following conclusions:

- the effect of variable A is similar to the intensification factor of the system,
- the effect of the variable B is equivalent to the time constant of the system,
- the effect of the variable C is strongly related to the nonlinear phenomenon described by the expression $C\varepsilon^2$. The variable C acts in the final part of the curve, and reacts in a contradictory way with the variable A. It gives a new dynamic to the system.

While comparing it with our model, we note that:

- the variable B acts in the first and second zone of the stress-strain curve, and
- the variables A and C act in the third zone.

It is important to note that the actions on the zones cited are independent. The factor T_0 defines an initial phase of the test and corresponds to the setting state in a progressive tension of fibres in the yarn. Vangheluwe's model appears to be incomplete, because it does not distinguish zones 1 and 2 from the stress-strain curve, i.e., the zone of fibre alignment and the zone of elastic linear strain.

Żurek's model

Żurek's model [7] is a relatively complete rheological model (Figure 5), because it confines the friction and inertia effects by the terms (*P*) and (*M*); combined together with the model of Kelvin-Voigt's (η, E_2), and supplemented in series by an

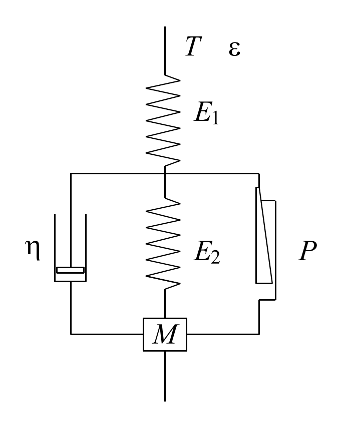

***Figure 5.** Żurek's model.*

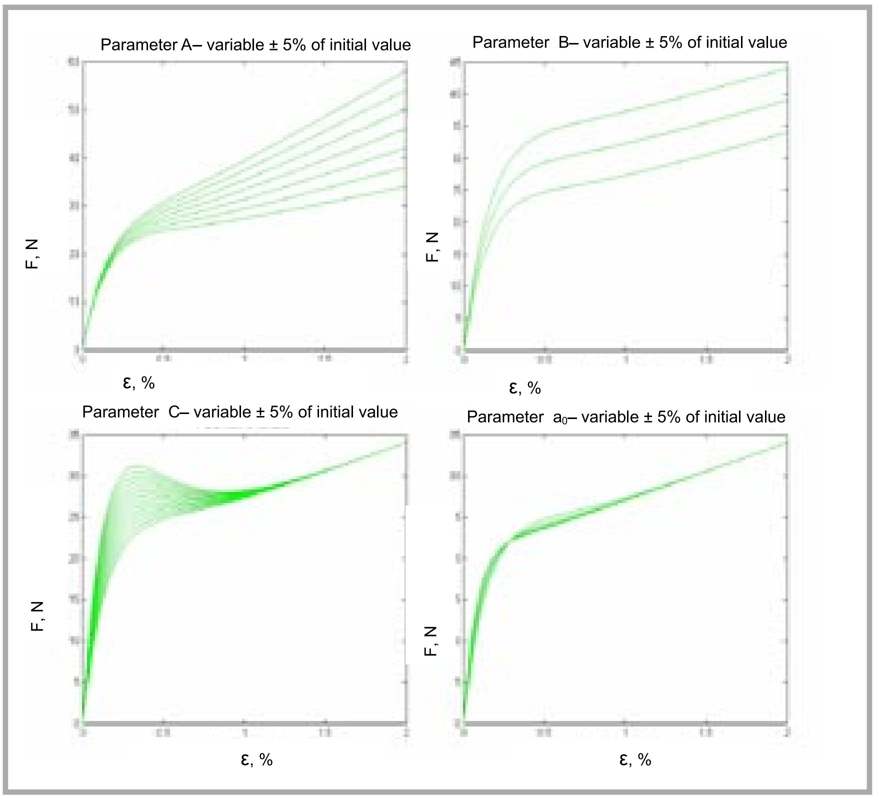

Figure 6. *Curves of sensitivity of the Żurek's model.*

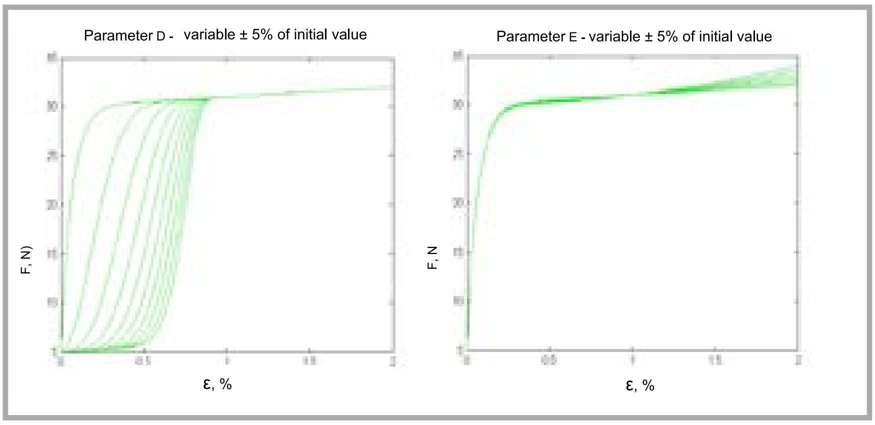

Figure 7. *Curves of sensitivity of the Vangheluwe's model modified by Manich.*

elasticity (E_1) assimilating the Hookean region.

The particular solution of the model is as follows:

$$T(\varepsilon) = A\varepsilon + B(C\varepsilon - B)exp(-a_0\varepsilon) \quad (10)$$

The particular dependencies between the parameters A, B, C, and α_0 of equation (10) and E_1, E_2, P, n, and M, as well as for Żurek's model modified by Manich (equation 12) are expressed in [6].

Sensitivity of Żurek's model

As before, only one variable was changed in order to determine its influence on the tensile force. Analysis of the curves in Figure 6 showed that:

- there is a variation of A similar to that of the parameter C of Vangheluwe's model with more noticeable linearity,
- the variation of B is regarded as an action on the static state of the system (static intensification factor),
- the variation of the variable *C* expresses the first realignment of the signal, which is visible in second order system, and
- the variation of "a_0" is associated with the time constant of the system.

Comparing it with our model, the parameters are analysed in the same perspective as in Vangheluwe's model; only the parameter *C* brings a specific dynamics at the end of the linear zone (between zone 2 and 3). This effect is noted only for certain yarns.

Manich's models

Manich [6] explained the nonlinearities differently. He used a different interpretation of Vangheluwe's and Żurek's models, in order to join the fibre phenomenon in the yarn. He proposed two modified models:

Vangheluwe's model modified by Manich

We analysed this model in a similar way. The difference between the basic model and the following modified model is expressed by the following equation:

$$T(\varepsilon) = T_0 + A[- exp(-B\varepsilon^D)] + C\varepsilon^E \quad (11)$$

acts only on the exponent of deformation terms. Also, we endeavoured to study only the complementary variables *D* and *E*.

The particular dependencies between parameters *A, B, C, D*, and *E* of equation (11) and physical qualities of the model are similar as expressed for equation (9).

Sensitivity of Vangheluwe's model modified by Manich

The curves in Figure 7 show us how these two terms influence the curve with the rest of the parameters unchanged.

The conclusions from the modifications are as follows:

- *D* plays an important role at the beginning of the curve. It introduces a horizontal bending at the beginning of the curve, which enables the time constant to be managed in a gentle way and an effect of delay to be introduced;
- E acts in the nonlinear part by giving a more constant dynamic at the end of the signal.

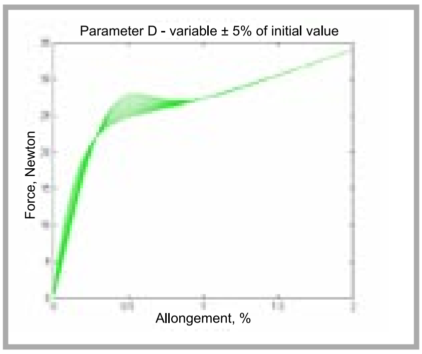

Figure 8. *Curves of sensitivity of the Żurek's model modified by Manich.*

Comparing it with our model, we note that:

- the variable D creates two zones 1 and 2, placed in one zone of the basic model, and
- E reflects the nonlinear effects only in zone 3.

Żurek's model modified by Manich

The modification made to Żurek's model is as follows:

$$T(\varepsilon) = A\varepsilon + B + (C\varepsilon - B)exp(-a\varepsilon^{D}) \quad (12)$$

The sensitivity of the model is presented in Figure 8:

The variation of *D* points out the effect of parameter *C* of the basic model, but in a way shifted in time, while acting temporavily in dependence of the temporary state of parameter *A*. This parameter combines carefully the effects of the parameters *a* and *C* at the basis model. As noticed when studying Vangheluwe's modified model, the parameter *D* also makes possible the existence of the two zones 1 and 2 of our model, but in a less noticeable manner at the beginning of the curve.

Legrand's model

This proposes an empirical model and describes traction including relaxation phenomena [7] as a system of Equations (13) in which each equation represents the curve phases.

Legrand finds that the velocity of extension and relaxation very significantly influences the shape of the traction curve. His approach is based on a study of a shape of the practical real curves.

To identify the parameters, it was necessary to make different tests:

1. a tensile test to obtain (a_1, b_1, c_1, d_1, e_1, f_1),
2. a relaxation test to obtain (*A, B, C, D*)
3. a hysteresis test to obtain (α, β, γ).

The part of Legrand's model for the traction phenomena taking into account the visco-elasticity concept is as follows:

$$T(\varepsilon) = a_1\varepsilon + b_1 + c_1 \sin(d_1(\varepsilon - e_1)exp^{f_1(\varepsilon - e_1)} \quad (14)$$

Sensitivity of Legrand's model

The curves of sensitivity of the particular parameters of the Legrand's model are presented in Figure 9.

The parametrical analysis leads us to the following conclusions:

- a_1 is the parameter which permits the management of the signal starting time. It adapts the slope of the continuous component;
- b_1 is positioned by an offset of sinusoidal component to the continuous component;
- c_1 manages the amplitude of the sinusoidal component;
- d_1 defines the period of the sinusoidal factor;
- e_1 represents the term of changing the phase of the sinusoidal component, and
- f_1 is a damping ratio of the sinusoidal component.

Summary

A new model for the yarn stress-strain curve was proposed. We tried to introduce the textile character of it by inter-

$$T(\varepsilon, \varepsilon^*, t) = \begin{cases} a_1\varepsilon + b_1 + c_1 \sin(d_1(\varepsilon - e_1)) \exp^{f_1(\varepsilon - e_1)} + A \exp^{Bt} + C \exp^{Dt} & when\ \varepsilon^* \geq 0 \\ F(\varepsilon) = \exp^{\alpha(\varepsilon - \beta)} - 1 & when\ \varepsilon^* < 0 \\ with\ \alpha = \gamma \ln(\varepsilon_1) + 1 & \end{cases}$$

Equations 13.

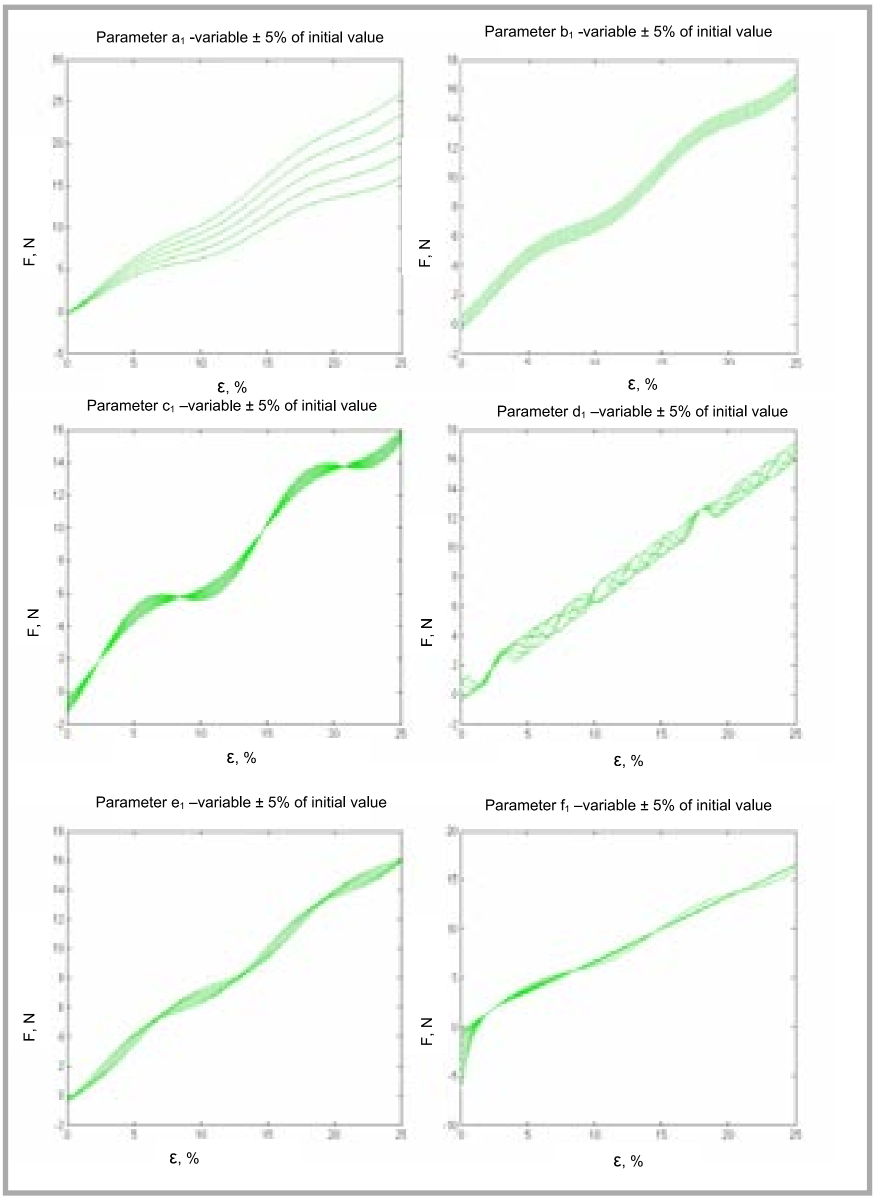

Figure 9. *Curves of sensitivity of Legrand's model (only one mentioned parameter is changed).*

preting this curve as a form of various models. This differentiation is due to a change in the yarn state during the stretching time. This model, then, differs very clearly from the other models described in the literature, because those models represent a different approach to the stretching problem. Also, in order to compare our work with those described in the bibliography, it was necessary to study the sensitivity of the parameters of the various models. The analysis of the curves of sensitivity enables us to show the influence of each parameter on the models and define their scope. As we can see, the scope of many parameters of the models described in bibliography is very large and extended onto two or three zones. In our model, however, the parameters influence only one zone which is precisely established; moreover, we are sure that it is exactly this parameter. We set up a strategy of development in order to avoid the effects of parametric compensation, which is very often observed, when the number of parameters which composed the model becomes large. Throughout the whole process, a 'stronger' parameter can cover up a a 'weaker' one. Our approach of dividing the model into sub-models contributes to the correct operation of the identification, because we use a method of identification by pieces with sub-models which have a small number of parameters. We believe that the models presented in the bibliography can lead to erroneous results, because of the significant number of parameters and their curve of sensitivity. The continuation of the study proceeded onto the identification of the unknown parameters of the model, which will be presented in the second part, to be entitled "Modelling the stress-strain curve of textile products: Part 2, Theoretical and simulated results".

Editorial note

This problem was presented at the Autex Conference 2003 in Gdynia, Poland.

References

1. E. Lucas; "Sur la géométrie du tissu", 1878.
2. Tchebishef; "Sur la coup des vętements", 1878.
3. L. Vangheluwe; "Study of the Time Dependent Mechanical Properties of yarns for Weaving" ; Doctoral thesis ; University of Ghent, 1992.
4. L. Vangheluwe ; "Relaxation and inverse relaxation of yarns after dynamic loading" ;Textile Research Journal ; p 552-556 ; N° 63(9), 1993.
5. W. Żurek, S. Aksan ; "A rheological model of viscose rayon" ; Journal Appl. Polym. Scien. ; p 3129-3137 ; N°19, 1975.
6. A. M. Manich. ; "Visco-elastic behaviour of polypropylene fibres" ; Textile Research Journal ; p 325-330 ; N°69(5) ; May 1999.
7. X. Legrand ; "Contribution ŕ la modélisation dynamique et ŕ l'identification de tissus techniques" ; L'université des sciences et technologies de Lille. Thesis, December 2000.
8. P. Bruniaux; "Contribution ŕ la modélisation non linéaire ŕ l'identification et ŕ la commande en boucle ouverte de moteur pas ŕ pas" ; Thčse de docteur de 3čme cycle, L'Université des Sciences et Technologie de Lille ; December 1988.

Received 12.05.2005 Reviewed 14.10.2005