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Method of Predicting the Pressure Drop on Woven Fabrics

Abstract
This article presents a mathematical model of pressure drop in woven fabrics, which is caused as the result of impact airflow through the product. The hypothetical time-dependency of the pressure obtained is considered as a reference pulse, and serves for comparing the real test of a textile product in relation to an accepted standard.

Key words: *air permeability, impact air permeability, mathematical model, rheology, woven fabrics.*

Introduction

Textile products (woven and knitted fabrics and nonwovens) may have very different applications, including for technical use. Textiles are used as safety measures for fire-brigades and safeguards in the form of protective clothing, fire-proof woven fabrics and other products, for example spring protectors. They are also universally applied as safety products in motor vehicles (for example as air bags and safety belts in cars), as well as air belts for aviation. Textiles are also connected with military equipment like parachutes, military clothing, protection against chemical, biological, and nuclear weapons, bullet-proof vests, and are also used in aircraft fuselages, and in spacetechnology, e.g. clothing used by astronauts in space flights.

Special attention should be paid to those textiles which are connected with conditions of dynamic use, as for example in the impact cushioning of an air bag or a parachute's canopy opening. Developing the use of such applications motivated investigation into the impact air permeability of flat textile products. Impact air permeability is a new product feature which describes the difference in behaviour of the product's surface measured under conditions of stationary and non-stationary through-airflows. The measure of impact air permeability is an index described in papers [1, 2, 3]. The impact air permeability index is a quotient of the difference between the area surfaces under the hypothetical (calculated) and the real (measured) pressure impulse in reference to the area under the hypothetical curve. The hypothetical pressure impulse is constructed, among others, with parameters obtained during static test of air permeability so tests of flat textile products are conducted in two stages: firstly under static conditions, and next under dynamic conditions. Under static conditions, a stationary airflow occurs through the porous structure, as represented by a woven fabric, and the shapes and dimensions of the product's pores do not change. The value of the volume-airflow w and the pressure p, which is equal to the pressure difference (pressure drop) occurring on both sides of the sample as a result of the airflow, are measured while the test is being carried out. Studies on the properties of textiles such as woven & knitted fabrics and nonwovens under such conditions are extensively described in [4, 5, 6], among other works. Next, the products are tested under conditions of a non-stationary airflow using a test stand specially designed for conducting such investigations [7, 8]. A brief scheme of the stand is presented in the next section (Figures 1 and 2). The following quantities are measured with the use of this stand: the deflection h(t) of the product, the displacement x(t) of the piston in the cylinder, which causes the flow of the air through the sample, the pressure difference (pressure drop) p(t) occurring on the product as a result of the piston's displacement, and the electric current i(t) of the electromagnet which forces the piston to move. All these quantities are measured as functions of time. On the basis of the static characteristics of the woven fabrics and their tests under dynamic conditions, the hypothetical pressure drop p'(t) on an ideal woven fabric is predicted, as the result of deduction, and on the assumption that the product's features will be the same, irrespective of the static or dynamic conditions of the test. A fabric which is manufactured from ideal elastic threads and has no mass is accepted as an ideal woven fabric. A lack of friction between the threads is also assumed. The pores of such a product return to their previous shape after the flow stops. The ideal woven fabric should behave similarly under both static and dynamic conditions. There is no doubt that such a product does not exist, and in reality a woven fabric is characterised not only by elasticity alone. During airflow, the fabric is subjected to instantaneous elastic, delayed, and durable deformations. The phenomena of creeping and relaxation occur, as does friction between the threads. This is the why the reference pulse never occurs on the tested product.

Mathematical model

One possible way of analysing the hypothetical pressure pulse is to use material reference models [1, 9]. A reference model is prepared in the form of a metal plate with appropriate formed orifices. The structure of such a model is stable and independent of the conditions. The points of the static characteristic of the material reference model are in the neighbourhood of those points which are characteristic of a textile product; these are taken as the reference points. In reality, the textile product is characterised by rheological features such as elasticity, viscosity, plasticity, and strength, as well as by its mass. The structure of the textile product also changes. It is difficult to determine the hypothetical pressure pulse by the use of non-deformable reference models because the model must be fitted to the tested sample of the textile product. This must be done in such a way that the flow characteristics of the model and the product are similar, considering the pressure values limited by the range of the changes in the volume airflow. Selecting the model's characteristic is very time-consuming. What is more, independently of the measurements carried out with textile products under static and dynamic conditions, additional measurements connected with reference models should be performed. Thus, the significantly increases for obtaining the goal in the form of the hypothetical pressure pulse. The justification of the use of reference models in the preliminary research which we carried out was the lack of tighness of the piston moving in the cylinder of the measuring stand for dynamical investigations, which was the predecessor of the stand currently being used. Thus, it was not possible to determine the time dependency of the volume airflow flowing through the barrier on the basis of testing the woven fabrics alone. The use of reference models was our solution to this problem while we carried out these early investigations. A new measuring stand for dynamic investigations was designed and constructed over the years 1999-2001 thanks to the research project grant [10]. Many practical corrections were introduced in comparison to the old model stand, but the most essential change was securing the tighness of the piston in the cylinder. Thanks to the design as accepted, it is now possible to determine the hypothetical pressure pulse on the textile product without the use of non-deformable reference models. Only the knowledge of the time dependency of the volume airflow w(t) is necessary. This was determined by analysing the changes to the geometrical parameters of the space which is formed between the fastened sample and the moving piston.

A simplified scheme of the measuring stand is shown in Figure 1. The stand has the shape of a cylinder. A tight piston with the diameter r moves inside the cylinder in a vertical direction, forced by an electromagnet. The piston motion is limited within the range of x_u to x_b.

The motion of the piston causes the air to flow through the porous barrier. The volume airflow which flows through the textile sample is given by the following equation:

$$w(t) = \frac{dV(t)}{dt}, \tag{1}$$

where:
V(t) is the volume of air in the cylinder limited by the surface of the sample tested and the moving piston.

Below we will consider the following cases related to the geometrical changes of the space volume between the barrier and the piston caused by its motion.

Case 1

The porous barrier is a non-deformable body, and is motionless over the period of the piston's displacement. The volume of the cylinder changes according to the following dependency:

$$V_w(t) = \pi r^2 x(t), \tag{2}$$

where:
r is the radius of the piston, and
x(t) is a function which describes the displacement of the piston in the cylinder, while the following inequality is fulfilled:

$$(x_d - x^*) \geq x(t) \geq 0$$

where:
x^* is the initial position of the piston.

By using Equation (2) for the volume airflow we obtain:

$$w(t) = \pi r^2 \frac{dx(t)}{dt}. \tag{3}$$

Case 2

The porous barrier, for example a woven fabric, is a deformable body, and is deformed over the piston's motion according to Figure (2)

The woven fabric is yielded and deflected over the piston's movement downwards. The shape which the woven fabric acquires is difficult to determine. Analysis of such a shape was carried out in [11]. For simplification, the assumption was made [12] that the woven fabric takes the shape of a spherical cup with the radius R(t). The volume of the spherical segment describes the expression:

$$V_k(t) = \pi h^2(t)[R(t) - \frac{1}{3}h(t)], \tag{4}$$

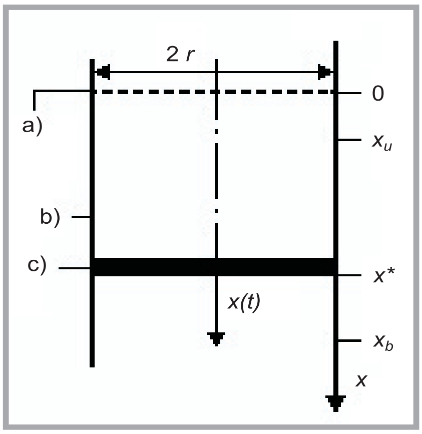

Figure 1. *A brief scheme of the measuring stand; a - porous barrier, b - cylinder, c - piston, x - the axis of the piston's position in the cylinder, 0 - position of the porous barrier; position of the piston: x_u - upper position, x* - initial position, x_b - botton position.*

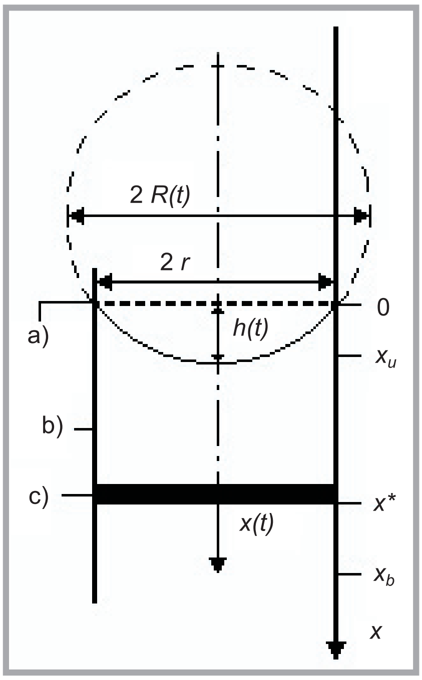

Figure 2. *Illustration of the case 2; a - porous barrier, b - cylinder, c - piston, x - the axis of the piston position in the cylinder, 0 - position of the porous barrier; position of the piston: x_u - upper position, x* - initial position, x_b - botton position.*

where:
$h(t)$ is the height of the spherical segment (the deflection of the sample), and

$$R(t) = \frac{r^2 + h^2(t)}{2h(t)}. \quad (5)$$

We can now state that, in the case considered, the volume of the space between the sample and the piston changes according to the following equation:

$$V(t) = V_w(t) - V_k(t). \quad (6)$$

By using Equations (2), (4), (5), and (6) we obtain

$$V(t) = \pi r^2 x(t) - \frac{\pi}{2} r^2 h(t) - \frac{\pi}{6} h^3(t). \quad (7)$$

Equations (3) and (7) can be used to find the dependency of the volume airflow as a function of time $w(t)$. After transformations, we obtain:

$$w(t) = \pi r^2 \frac{dx(t)}{dt} - \frac{\pi}{2} r^2 \frac{dh(t)}{dt} - \frac{\pi}{6} \frac{dh^3(t)}{dt}. \quad (8)$$

As

$$\frac{dh^3(t)}{dt} = 3h^2(t) \frac{dh(t)}{dt} \quad (9)$$

we finally obtain:

$$w(t) = \pi r^2 \frac{dx(t)}{dt} - \frac{\pi}{2} [r^2 + h^2(t)] \frac{dh(t)}{dt} \quad (10)$$

A Maxwell structural rheological model [13] was proposed to elaborate the behaviour description of the deformable body formed by the porous barrier placed on the upper base of the cylinder, and to determine the dependencies between the stresses and strains (deformations) of the barrier. The model consists of two mechanical models, the ideal spring (the Hooke's solid) and the piston with orifices (the Newtonian fluid) connected in series.

During the impact airflow through the woven fabric, the product is deformed. It was assumed that the elasticity force which arises at this moment is proportional to the force of deformation. The measure of the deformation of the porous barrier is the ratio of the absolute deformation ($d - d_0$) and the initial value d_0, which characterises the shape of the rheological body analysed. The relation between the stress σ_A and the deformation of the body is given by Hook's law:

$$\sigma_A = E \frac{d - d_o}{d_o}, \quad (11)$$

where E is the Young modulus in N/m^2

In the case of the woven fabric, it was accepted that d_0 consists of the measuring area $A = \pi r^2$, whereas d is the area of the spherical cup (the sample's surface) $S_k = \pi(r^2 + h^2)$ formed by the woven fabric as a result of the airflow. Thus, Equation (11) takes the form of:

$$\sigma_A = E \frac{h^2(t)}{r^2}. \quad (12)$$

The equation of the state of the mechanical model in the form of a piston with orifices is given by:

$$\sigma_B = k_x \frac{d\varepsilon(t)}{dt}, \quad (13)$$

where
$\varepsilon(t)$ is the relative displacement of the piston in relation to the cylinder, e.g. the relative change of the co-ordinate which describes the position of the piston in the cylinder, and
k_x is the proportionality coefficient (the viscosity modulus) in Ns/m^2.

For the Maxwell model,

$$\sigma_A = \sigma_B. \quad (14)$$

Considering Equations (12) and (13), equality (14) takes the following form:

$$E \frac{h^2(t)}{r^2} = k_x \frac{d\varepsilon(t)}{dt}. \quad (15)$$

The path moved by the piston from the initial position x^* to the position x_d is given by:

$$x_z(t) = \frac{V(t)}{\pi r^2}, \quad (16)$$

where the function $V(t)$ is not given by the volume of the cylinder, but by the dependency (7) which takes the deformation of the product under consideration. Thus, we can obtain the relative change of the co-ordinate, which describes the position of the product in the cylinder, and is equal to:

$$\varepsilon(t) = \frac{x_z(t)}{x^*}, \quad (17)$$

Furthermore, the initial piston position can be given by:

$$x^* = \frac{V_o}{\pi r^2}, \quad (18)$$

where:
V_0 is the initial volume of air which fills the cylinder, as limited by the deformable product's sample and the piston placed at the initial position x^* (the volume of the initial chamber).

Considering the expressions (16), (17), (18), and (1), Equation (15) becomes the form:

$$E \frac{h^2(t)}{r^2} = \frac{k_x}{V_o} w(t). \quad (19)$$

The normal stresses σ_A and σ_B occurring at stretching and compression (among other times), and which determine the value of the force acting on an area unit of the element's cross-section, are considered as equivalent with the pressure p'. Therefore, Equation (19) can be presented in the form of the two following dependencies:

$$p'(t) = E \frac{h^2(t)}{r^2}; \quad (20)$$

and

$$p'(t) = \frac{k_x}{V_o} w(t). \quad (21)$$

Therefore, the problem of finding the hypothetical pressure pulse comes down to determining the value of E (by using Equation 20) or the quotient k_a/σ_a (by using Equation 21). Determining the constant E, which is connected only with the elasticity features of the woven fabric, is difficult. For an ideal woven fabric, the assumption is made that the fabric is manufactured of ideal elastic threads, which would have no mass. The lack of friction between the threads is also assumed. The pores of such a product return to their previous shape after the flow stops. In reality, a woven fabric is characterised not by elasticity alone. During the airflow through the fabric, it is subjected to deformations, which are instantaneous, elastic, delayed, and durable. The phenomena of creeping and relaxation also occur. This was why we focussed our attention on Equation (21), which describes the hypothetical pressure pulse on the woven fabric as well as Equation (20) does. The following consideration was carried out with the aim of determining the value of the proportionality coefficient.

It was assumed that each value of the volume airflow flowing through the product corresponds to a value of the pressure p', independently of how the airflow intensity is determined. Assuming the ideal elastic features of the woven fabric, we can state that its behaviour under static ($w \neq f(t)$) and dynamic ($w = f(t)$) conditions should be similar. Considering the above-mentioned assumption, we obtain the following dependency (function composition):

$$p'(w(t)) = p_s(w(t)). \quad (22)$$

and further:

$$p'(t) = ps(w(t)). \quad (23)$$

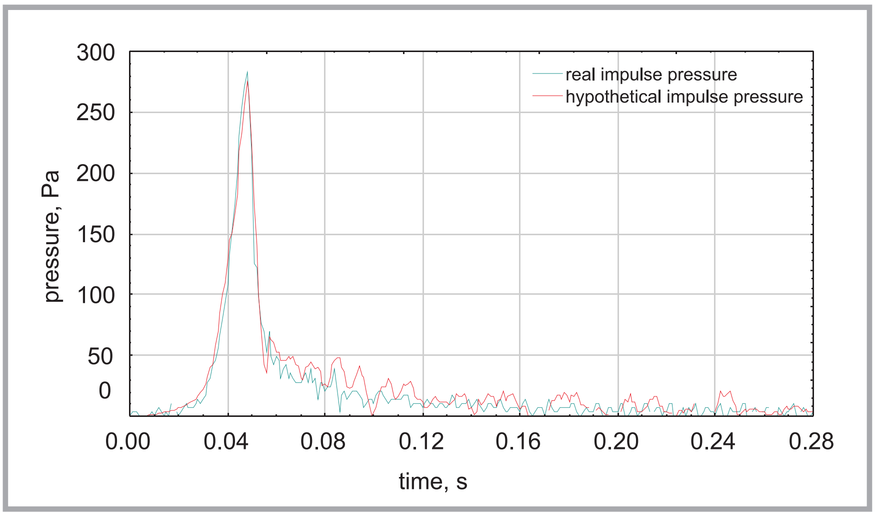

Figure 3. *Hypothetical and real pressure pulses obtained for a cotton woven fabric.*

It is already known that under static conditions the dependency of pressure on the volume-airflow $p_s(w)$ is a linear relation without a free term in the expression.

Then we can state:

$$p_s(w) = aw. \tag{24}$$

The dependency (24) can also be written in the following form:

$$p_s\,(w(t)) = aw(t). \tag{25}$$

From (23) and (25) we have:

$$p'(t) = aw(t). \tag{26}$$

On the basis of Equations (21) and (25), the proportionality coefficient k_x/V_0 can be determined as:

$$\frac{k_x}{V_o} = a\,. \tag{27}$$

Considering the expressions (10) and (27), Equation (21) finally takes the form:

$$p'(t) = a[\pi r^2 \frac{dx(t)}{dt} - \frac{\pi}{2}[r^2 + h^2(t)]\frac{dh(t)}{dt}]. \tag{28}$$

The equation presented above describes the hypothetical pressure difference (pressure drop) of the air, which occurs on a deformable porous barrier in the form of a flat textile product, over impact airflow caused by a single, impact piston movement. This pulse is considered as the reference pulse, and serves to estimate the behaviour of a real woven fabric. The comparison of the behaviour of a textile product sample, for example a woven fabric, in relation to an accepted standard, depends on the confrontation of two time-dependencies of the reference pulse *p'(t)*: one which never occurs on the product tested, and the real pulse *p(t)*. A difference between these dependencies certifies that the woven fabric has in reality not only elastic properties, but is characterised by all rheological features, such as elasticity, viscosity, plasticity, and strength, as well as by mass. All these features mean that the changes in the pore dimensions, under the influence of pressure, differ in the real fabric from those for an ideal elastic body.

As an example, Figure 3 presents two pressure time dependencies, the real and the hypothetical, calculated on the basis of the expression (28) for a cotton woven fabric with the following structural parameters: twill weave, thickness – 0.72mm, area mass – 254g/m^2, apparent density – 353kg/m^3 (calculated as a quotient of the area mass over the thickness), number of warp threads – 35 per cm, number of weft threads – 19 per cm, linear density of warp – 43.0 tex, and linear density of weft – 54.3 tex. All threads were without twist.

The differences between the real and the hypothetical impulse pressures (Figure 3) means that the sample of a flat textile product behaves in a different manner during static and dynamic air flows. Besides, the maximum volume of the real impulse pressure is less than the maximum volume of the hypothetical impulse pressure. This means that the shapes and dimensions of the woven fabric's pores create easy a higher resistance to the air flow during the transient state compared with stationary conditions.

Conclusions

1. The hypothetical pressure pulse considered as a reference pulse serves to compare the behaviour of a woven fabric sample in relation to an accepted standard, e.g. a real woven fabric.
2. The knowledge of the hypothetical and the real pressure pulses allows us to calculate the index of the impact air permeability. This index is a measure which allows products to be scheduled while considering their dynamic flow properties.

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