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An Analysis of the Complex State of the Stresses in Paper as Exemplified with Bursting Test

Abstract
Among the research works that have been carried out in the Institute of Papermaking and Printing of the Technical University of Łódź, an attempt has been made to describe the complex state of the in-plane stresses of paper. The paper used for the bursting test has been considered as a thin film in which the existing stresses and strains have been evaluated. The material constants have been evaluated on the basis of the results of the uniaxial stress. The calculations of the bursting pressure and strains of the specimen being burst have been carried out using the selected hypotheses on strength properties, considering paper as an elastic or viscoelastic body. The obtained results of the calculations have been compared with the results of the measurements in order to verify the accepted assumptions and the methodology of the calculations.

Key words: *burst, bursting test, paper strength at the complex states of the stresses, Poisson ratio, hypotheses on strength properties, elastic strain, viscoelastic strain, shear stress.*

Introduction

Paper is a material which indicates properties of viscoelastic bodies; it is characterised by the anisotropy of its strength properties. Moreover, it is a material which shows a heterogeneous structure. These properties mean that a description of the behaviour under different stress states is very difficult, and no rheological model exists which could fully reflect the behaviour of paper. For that reason, in order to be able to define any given paper, different rheological models are selected depending on individual requirements [1].

In many cases, paper may be considered as an elastic body. Obviously it has to meet the certain conditions. It is common knowledge that in stressed paper, it is possible to distinguish three types of strains: elastic strains, which disappear after removal of the stress; those viscoelastic strains which after removal of the stress decrease with passage of time, and those viscoelastic strains which remain unchanged after removal of the stress. In many cases, such as short-period loads or loads which cause the creation of very small strains, the dominant role is played by the elastic strains. In this case, one can consider paper as an elastic body without making a serious error. In the other cases, when other kinds of strain reach significant values, then it is possible to apply a description that is based on rheological models. Very convenient in use are the four parameter models, such as Buerger's model or its equivalent, the double Maxwell model. As a matter of fact they do not reflect some phenomena, e.g. the strengthening (consolidation) of paper, but they permit a relatively easy description of many of its basic behaviours.

The majority of machine-made papers shows the properties of the orthotropic bodies [2-4]. They have distinctly different tensile strengths at stretching, both in machine and cross direction, which additionally complicates the description of their behaviour when subject to the loads. The above mentioned characteristic of paper decides the way it is tested as well as the description of its properties.

In practice, the strength properties are tested in different loading situations in order to determine such properties as breaking length, stretchability, bursting strength, tearing resistance, stiffness and others. On the basis of the results of such tests, the suitability of the papers and boards for their practical use is evaluated.

Analytical or numerical methods, which would permit the selection of the materials and the design of the paper products to be optimised, and would thus lead to a decrease in their design and manufacturing costs, are rarely used.

During the processes of paper conversion or daily use, the paper is most often subjected to uni- or multidirectional stresses in the plane. During the unidirectional stresses, it is enough to perform the unidirectional tensile test to evaluate the strength properties. During the complex state of the stresses in-plane, the strength of paper may be estimated on the basis of the bursting test, among others. In this test, however, only the state of stresses is tested, which sometimes does not correspond with the facts that arise in reality.

Sometimes a comparison of the bursting and tensile tests is carried out by comparing a strip of paper subjected to unidirectional stretching with an analogous strip cut from the paper tested for the bursting strength, the axis of which is the diameter of the specimen being burst. Such simplified reasoning is burdened with error, in result of different states of the stresses under which both specimens were.

Any more profound analysis of the bidirectional state of the stresses requires the Poisson ratio to be known; further, in the case of the destructive test, the proper hypothesis for strength properties must also be accepted. In practice, it turned out that in application to the various materials, the different hypotheses for strength properties prove to be true.

According to the hypothesis of the greatest shear stresses, the strength of material depends on its shear resistance; the reason for the destruction of the test piece is the appearance of shear modules which exceed the admissible value [5]. In practice, the use of this hypothesis amounts to determining the greatest shear stresses and comparing them with the permissible stresses. If between the main stresses σ_x, σ_y, σ_z there occurs a relationship described with formula (1), then the value of the peak shear stresses τ_{max} may be described using formula (2)

$$\sigma_x > \sigma_y > \sigma_z \quad (1)$$

$$\tau_{max} = \frac{\sigma_x - \sigma_z}{2} \quad (2)$$

The above reasoning is proved in relation to such materials as steels. It is also confirmed in the crystal structure, where permanent strains occur as a result of slides or twinnings. In order to achieve this, shear stresses must occur.

The strains in paper cause the inter-fibre bonds to break, and also the fibres break and change their position in the paper structure. While analysing the paper structure and the directions of its strain, it is difficult to connect the failure process of this material with the occurrence of the maximal shear stresses in certain planes. It seems more reasonable to connect the fact of the material failure with an excess of permissible size of the relative strains, and thus the use of the hypothesis of the peak strain rates seems to be more justified.

Experimental

In the Institute of Papermaking and Printing at the Technical University of Łódź, research has been carried out on the basis of a comparison of the bursting pressures and the convexity of the specimens being burst that were obtained during the measurements with the theoretically obtained values. In order to simplify the calculations, the research works were performed using filter papers which are characterised with an isotropy of the strength properties in plane. The analysis was carried out for the biaxial in-plane state of stresses, considering the paper as a thin film subject to biaxial stretching. It has been assumed that neglecting the changes in the thickness of paper which was in-plane stressed does not affect the test results because of the porous structure of the paper.

The burst test according to standard PN-ISO 2758 was carried out; however, the stretch tests were based on standard PN-EN ISO 1924-2 with the deviation from the rule consisting in samples of 20 mm width, not provided for the test of the stretch velocity made not in conformity with the standard. For each kind of paper and testing (stretching, double stretching and break resistance) to receive the needed load times, different rates of strain were selected. For paper with grammage of 105 g/m^2, rates of 2, 0.2, and 0.02 cm/min were used. For other papers the rates of 5, 0.5, and 0.05 cm/min were applied.

The material constants were estimated on the basis of testing the unidirectional tensile strength of paper. In order to avoid the errors which could be caused because of the different velocities of the strains in the tensile and bursting strength tests, the velocities were selected in such a way that the times for achieving the failure of the specimen were similar. In order to determine the participation of the stable strains existing in the papers that were subjected to an uniaxial stress, these were tensioned to up to 90% of their tensile strength, and then after removal of the stress we waited for as long as was necessary for the relaxation of the viscoelastic strains; afterwards, the specimen was stressed again. The tests were carried out for the following three different times of load acting: 3, 30 and 300 s.

The measurements of the Poisson ratios were carried out using a non-contact technique consisting of the photographic registration of the strains during the uniaxial stretching of the paper [6,7].

During the tests used for estimating the bursting pressure, the height of the convexity of the burst specimen was also determined. According to the standard, the results of the ten tests as an average value have been calculated.

On the basis of theoretical calculations using the digitising method in the segments, the bursting pressure and the height of the burst specimen convexity were estimated. In order to perform the calculations, the burst convex specimen was divided by the planes which were parallel to the plane of the specimen before its failure. The fragments of the specimen convex between the individual planes were simplified to a shape of the conical surfaces, as shown in Figure 1.

For calculation purposes, it was assumed that in each segment the circumferential stresses σ_{s_i} have a constant value, and the radial stresses σ_{r_i} are inclined at an angle determined by the generating line (generatrix) of the next segment. The acceptance of such simplifications upon simultaneous assumption of the high amount of segments with small magnitude causes the effect of the simplifications on the results of the calculations to be insignificant. From the equations for the equilibrium of forces for the *Y* and *X* axes, the dependencies describing the radial and circumferential stresses and strains have been determined. On the basis of these relationships, calculations have been carried out in which a paper was considered as an elastic material, the behaviour of which has been described by means of Buerger's model. In the calculations two hypotheses concerning the strength properties were used, namely the hypothesis of the highest shear stress and the hypothesis of the highest strains. Additionally, the height of the burst specimen convex was also calculated using a simplified method by calculating the elongation of the burst specimen along its diameter, assuming that it was uniaxially stretched.

In the tests, the filter paper of the grammages 83 g/m^2, 105 g/m^2 and 162 g/m^2 was used. Part of the specimens of 83 g/m^2 grammage was calendered at a low linear load in order to consolidate the paper structure. After this calendering, the thickness of the specimens decreased more than twice.

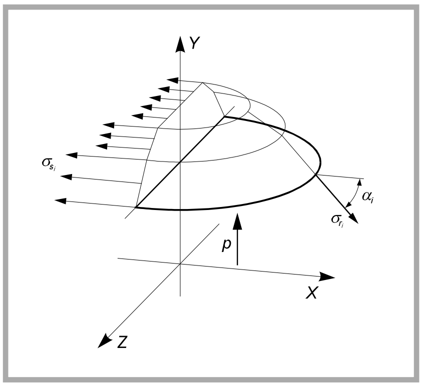

Figure 1. *Schematic diagram of an outline of the specimen convex divided into segments.*

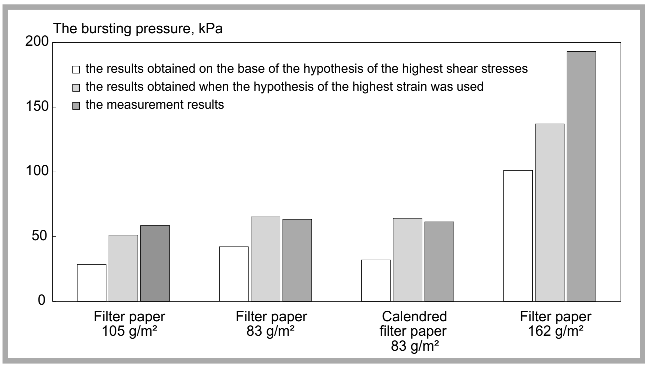

Figure 2. *The comparison of the results of the measurements and calculations of the bursting pressure made upon an assumption of the viscoelasticity of the tested materials.*

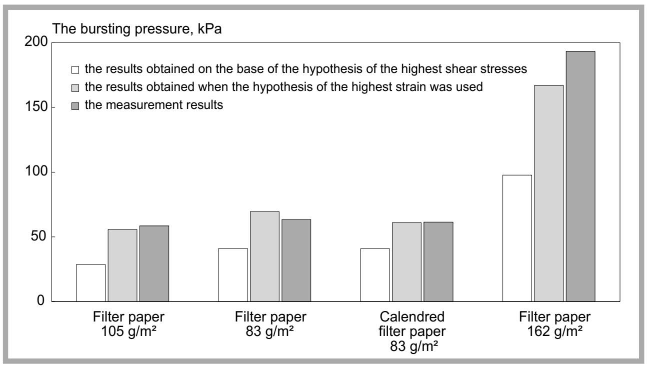

Figure 3. *The comparison of the results of the measurements and calculations of the bursting pressure made upon an assumption of the elasticity of the tested materials.*

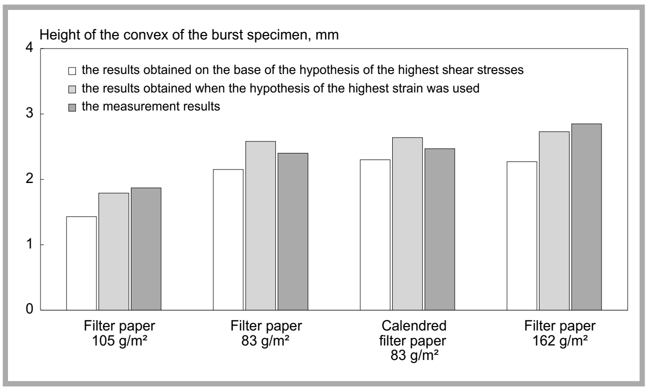

Figure 4. *The comparison of the measurement and calculation results of the heights of the convex specimens at the moment of failure.*

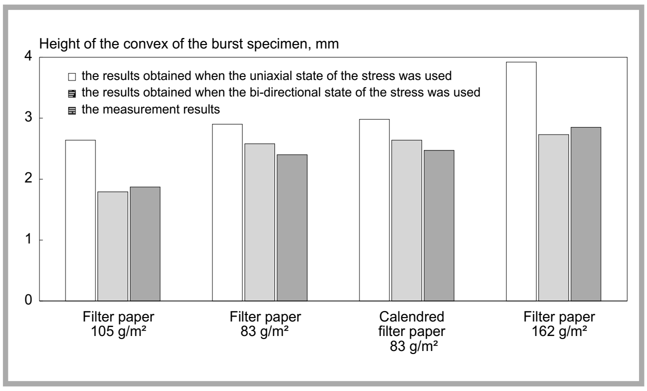

Figure 5. *The comparison of the measurement and calculation results of the heights of the specimen convexity at the moment of failure.*

The Research Results

At the conclusion of the performed experimental studies, it was found that in the papers being tested for elongation over 3 s, the participation of the permanent set in the total strain amounted to 5.7% up to 7.3%. The shares of the permanent sets obtained after 30 s and 300 s amounted relatively to between 15% and 22.7%, and between 26.5% and 50.8%. As the failure of the burst specimen took place within the time range of 2.8 s to 4.1 s, one can assume that in bursting strength testing, the share of the permanent sets was very small.

Figures 2 and 3 illustrate the comparison of the measurement results and the calculations of the bursting pressure obtained when the hypothesis of the highest shear stresses and the highest strains was used. The first diagram presents the bursting pressures calculated upon an assumption of the viscoelasticity of the papers; on the other hand, the second diagram shows the pressures calculated upon the assumption that the materials are elastic. The error rate of the bursting pressure measurements fall within the range of 1.5% to 3.3%, and are marked on the diagrams. The error rate of the measurements of the height of the convex specimens at the moment of their failure fall within the range of 0.5% to 3.0%, and are also marked on the diagrams. The error rate of burst pressure calculation results made on the basis of the greatest shear stresses hypothesis fall within the range of 0.7% to 1.3%, but the calculations made on the base of the strength properties hypothesis fall within the range of 0.3% to 1.1%.

Regardless of the assumption that paper is considered as viscoelastic or elastic, all the results of calculations carried out using the hypothesis of the highest shear stresses differ considerably from the real values, and are lower than them. On the other hand, the use of the highest strain hypothesis gives much better results and permits the exact calculations of the bursting pressure.

On the basis of the results analysis presented on the diagrams, it can be stated that use of the viscoelasticity theory as well as of the theory of elasticity gives a similar precision of the bursting pressure calculations.

The results of the measurements and calculations of the height of the specimen convexities by use of the hypothesis of the greatest stresses are shown in Figure 4. It shows that for all the materials tested, the heights of the specimen convexities that were calculated by use of the greatest strains hypothesis are closer to the real values than the heights obtained using the hypothesis of the greatest shear stresses.

In Figure 5, the results of calculation of the convexity heights at the moment of the specimen failure with the assumption of the uniaxial and multiaxial state of stresses. Matching up the results, we can state that an attitude assuming an uniaxial state of the stresses in the paper bursting test gives results that are burdened with significant errors (on average, they are higher by about 30% in relation to the real values). The calculation results obtained at an assumption of the complex state of the stresses are close to the real values, and their average error amounts to about 5%.

Conclusions

- On the basis of the research results obtained, it was found that in the destructive tests of short duration characterised by small participation of the permanent sets, it is permissible in the engineering calculations to regard the paper as an elastic body.
- During analysis of the strains, stresses and the strength property of paper, the velocity of the material strain is very significant, because it has a decisive influence for the tested case the paper can be considered as a viscoelastic or an elastic body.
- During the strength property analysis of the paper being subjected to the destructive tests of short duration, during which complex states of stresses appear, the best results are obtained when the hypothesis of the highest strains is used.
- The calculations of the convexity height of the burst specimen confirm the thesis that the simplified analysis of the bursting strength test based on an uniaxial state of stresses is burdened with a significant error.
- The thickness changes of the paper (if they do not destroy the fibres) have little effect on its strength property, as they take place mainly at the cost of the decrease in the free spaces among the fibres. This confirms the correctness of regarding the papers stressed with forces acting in the plane of papers as in thin films, for which changes in thickness are not taken into consideration.

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