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Optimal Design of Reinforcing Fibres in Multilayer Composites using Genetic Algorithms

Abstract
The present paper is devoted to the problem of optimally design reinforcing fibres in a composite so that the material satisfies requirements over a range of mechanical properties. A method based on the genetic algorithm has been used in the optimisation process. A mathematical model of the fibre-reinforced multilayer composite and the conditions for optimal design of fibre shape and orientation are set forth in this paper. A description of the simple genetic algorithm applied to the optimisation of a composite structure is also presented. The problem considered in the paper is illustrated by a simple example.

Key words: *optimisation process, genetic algorithm, fibre-reinforced multilayer composite, mathematical model, fibre shape and orientation, stiffness of structure.*

Introduction

Fibre-reinforced composites are modern construction materials from which products used in many areas of technical applications are made. These materials are characterised by very good mechanical properties. They are ideal for structural applications where high strength and stiffness are required. The mechanical properties of the composite are not only defined by the properties of reinforcing fibres and their percentage participation in this material; the full advantages of such materials are obtained when the fibres are optimally distributed and oriented in each layer, with respect to the assumed objective behavioural measure in the optimisation process under the structure's actual loading conditions.

The optimal design of a structure made of a composite material, particularly regarding the shape and orientation of the reinforcing fibres, has been the subject of many scientific papers, including in [1-6]. Different methods of solving the optimisation problem were presented, but most of them were based on the gradient-oriented search algorithm.

The other approach, based on imitation of the evolution processes proceeding in nature, has also been used in the optimisation of structural components for many years. This method is known as the genetic algorithm. Its use has found growing interest in engineering design problems. The genetic algorithm is a simple, powerful and effective tool used for finding the best solution in a complicated space of design parameters. This algorithm is very different from traditional optimisation methods. The genetic algorithm only needs the information based on the objective function, which is its main advantage in comparison with methods based on information gained from objective function derivatives. In contrast to gradient methods, which often fall into a local optimum, this algorithm always finds the global optimum. Thus, the genetic algorithm can serve as an alternative method to classic methods based on mathematical programming. It can be used in many fields where the optimisation process is necessary, particularly in the optimal design of a structure made of a fibre-reinforced multilayer composite.

Model of Fibre-Reinforced Multilayer Composite

Optimisation is an essential process when designing a structural component in engineering practice. First of all, before this process, a physical and mathematical model of structure is built. This model is a starting point to formulating the optimisation problem.

In the paper, the problem of optimal design of reinforcing fibres in a composite material is considered using the simple model presented, for instance, in [7]. The purpose of the modelling process is to determine the extensional stiffness matrix for the multilayer composite (Figure 1), and to express its components in terms of design parameters and engineering constants of fibre and matrix materials.

The modelling process of the multilayer composite is based on the following assumptions [7]:

- The composite is a laminate made of a stack of permanently combined layers. The joint surfaces are infinitesimally thin, and they do not permit interlayer shearing.
- The layers are symmetric in geometry and material properties about the middle plane of the composite.
- The multilayer composite is a homogeneous material on a macroscale level, but its properties depend in turn on the properties of the layers.
- Each layer is a lamina made of a matrix reinforced with a ply of unidirectional fibres, and has a thickness h_k (the lamina thickness is very small compared to its length and width). The bonds between fibres and matrix are perfect.
- The lamina is macroscopically homogeneous, orthotropic and linearly elastic.
- The reinforcing fibres are homogeneous, isotropic, linearly elastic, regularly spaced and perfectly aligned in each layer. Young's modulus and Poisson's ratio of the fibre in the k-th layer of the composite are denoted by $E_w{}^k$ and $\nu_w{}^k$, respectively. The fibre density for each particular layer (volume of the fibres in the k-th layer / total volume of the k-th layer) is denoted by $\rho_w{}^k$.
- The matrix is homogeneous, isotropic and linearly elastic. Young's modulus and Poisson's ratio of the matrix in the k-th layer of the composite are denoted by $E_m{}^k$ and $\nu_m{}^k$, respectively, and its density $\rho_m{}^k$=1-$\rho_w{}^k$.

The extensional stiffness matrix for the homogeneous model of the multilayer composite

In conformity with the above assumptions, each particular layer of the composite is treated on a macroscale level as a plane, homogeneous and orthotropic material (Figure 1a). Using the rule of mixtures, the engineering constants of the k-th layer can be expressed in terms of the mechanical properties of the fibres and the matrix, and in terms of fibre density for this layer. They have the following form [7]:

$$
\begin{aligned}
E_1^k &= E_w^k \rho_w^k + E_m^k (1-\rho_w^k) \\
E_2^k &= \frac{E_w^k E_m^k}{E_m^k \rho_w^k + E_w^k (1-\rho_w^k)} \\
\nu_{12}^k &= \nu_w^k \rho_w^k + \nu_m^k (1-\rho_w^k) \\
G_{12}^k &= \frac{G_m^k G_w^k}{G_m^k \rho_w^k + G_w^k (1-\rho_w^k)}
\end{aligned}
\qquad (1)
$$

The notations E_1^k and E_2^k are the apparent Young's moduli in the fibre direction and in the direction transverse to the fibres, respectively, while ν_{12}^k is the so-called major Poisson's ratio, and G_{12}^k denotes the in-plane shear modulus in the k-th layer of the composite.

For an orthotropic material in co-ordinates aligned with principal material axes 1-2, coinciding with the fibre direction and the direction perpendicular to the fibre, the strain-stress relations in terms of the engineering constants are defined thus:

$$
\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1^k} & -\frac{\nu_{21}^k}{E_2^k} & 0 \\ -\frac{\nu_{12}^k}{E_1^k} & \frac{1}{E_2^k} & 0 \\ 0 & 0 & \frac{1}{G_{12}^k} \end{bmatrix} * \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \qquad (2)
$$

where ε_1, ε_2, γ_{12} and σ_1, σ_2, τ_{12} are strain and stress components with respect to these axes, respectively.

On the other hand, the stress-strain relations for an orthotropic layer are given by:

$$
\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11}^k & C_{12}^k & 0 \\ C_{21}^k & C_{22}^k & 0 \\ 0 & 0 & C_{33}^k \end{bmatrix} * \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \qquad (3)
$$

where $\mathbf{C}^k$ denotes the stiffness matrix for the k-th layer of the composite with respect to the material axes 1-2. This matrix is obtained by inversion of the compliance matrix appearing in equations (2). Thus, the nonzero components of $\mathbf{C}^k$ take the form of equation 4, and are explicitly expressed in terms of the engineering constants for the k-th layer of the composite material.

$$
\begin{aligned}
C_{11}^k &= \frac{E_1^k}{1-\nu_{12}^k \nu_{21}^k} \\
C_{22}^k &= \frac{E_2^k}{1-\nu_{12}^k \nu_{21}^k} \\
C_{12}^k = C_{21}^k &= \frac{\nu_{12}^k E_2^k}{1-\nu_{12}^k \nu_{21}^k} \\
C_{33}^k &= G_{12}^k
\end{aligned}
\qquad (4)
$$

The stresses and strains appearing in equations (2) and (3) were defined in the principal material directions for an orthotropic material. However, a composite material is often constructed in such a manner that its principal material directions do not coincide with a co-ordinate system that is geometrically natural to the solution of the problem. In this case, the stress-strain relations for an orthotropic lamina in the global coordinate system are:

$$
\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{C}_{11}^k & \overline{C}_{12}^k & \overline{C}_{13}^k \\ \overline{C}_{21}^k & \overline{C}_{22}^k & \overline{C}_{23}^k \\ \overline{C}_{31}^k & \overline{C}_{32}^k & \overline{C}_{33}^k \end{bmatrix} * \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \qquad (5)
$$

where $\overline{\mathbf{C}}^k$ denotes the stiffness matrix for the k-th layer of the composite with respect to the global reference system x-y, while σ_x, σ_y, τ_{xy} and ε_x, ε_y, γ_{xy} are stress and strain components with respect to the axes of this system, respectively.

Denoting by θ_k the angle between the fibre ply direction at a given point of the k-th layer and the x-axis of the global co-ordinate system (Figure 1a), the components of $\overline{\mathbf{C}}^k$ which are given by [7] are presented in equation (6) and follow from the transformation rule associated with the rotation of the co-ordinate system.

As shown in Figure 1, the fibre-reinforced multilayer composite is modelled by a homogeneous and orthotropic material. Using the classical lamination theory [7] for such a defined model, the generalised plane stress and strain fields are interrelated by Hooke's law of the form:

$$
\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} * \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \qquad (7)
$$

where $\mathbf{D}$ denotes the extensional stiffness matrix for the model of the multilayer composite. The components of this matrix are expressed by:

$$
D_{ij} = h_o * \overline{C}_{ij}^{o} + 2\, h_k * \sum_{k=1}^{n} \overline{C}_{ij}^{k} \qquad (8)
$$

where the coefficients $\overline{C}_{ij}^k$ follow from equation (6), and the summation is expanded over all layers of the composite material.

$$
\begin{aligned}
&\overline{C}_{ij}^k = \overline{C}_{ji}^k \qquad and \qquad i, j = 1,2,3 \\
&\overline{C}_{11}^k = C_{11}^k \cos^4\theta_k + 2(C_{12}^k + 2C_{33}^k)\sin^2\theta_k \cos^2\theta_k + C_{22}^k \sin^4\theta_k \\
&\overline{C}_{12}^k = (C_{11}^k + C_{22}^k - 4C_{33}^k)\sin^2\theta_k \cos^2\theta_k + C_{12}^k(\sin^4\theta_k + \cos^4\theta_k) \\
&\overline{C}_{13}^k = (C_{11}^k - C_{12}^k - 2C_{33}^k)\sin\theta_k \cos^3\theta_k + (C_{12}^k - C_{22}^k + 2C_{33}^k)\sin^3\theta_k \cos\theta_k \\
&\overline{C}_{22}^k = C_{11}^k \sin^4\theta_k + 2(C_{12}^k + 2C_{33}^k)\sin^2\theta_k \cos^2\theta_k + C_{22}^k \cos^4\theta_k \\
&\overline{C}_{23}^k = (C_{11}^k - C_{12}^k - 2C_{33}^k)\sin^3\theta_k \cos\theta_k + (C_{12}^k - C_{22}^k + 2C_{33}^k)\sin\theta_k \cos^3\theta_k \\
&\overline{C}_{33}^k = (C_{11}^k + C_{22}^k - 2C_{12}^k - 2C_{33}^k)\sin^2\theta_k \cos^2\theta_k + C_{33}^k(\sin^4\theta_k + \cos^4\theta_k)
\end{aligned}
$$

Equation 6

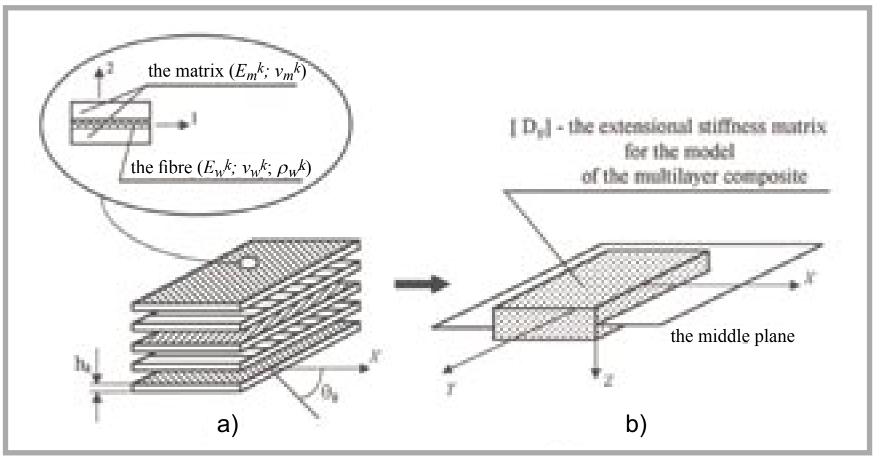

Figure 1. *Fibre reinforced multilayer composite; a) the real composite, b) the model of the composite.*

In addition to the model of the fibre-reinforced multilayer composite presented above, more complicated and more accurate models of a laminate exist. However, the generalised stiffness matrix for these models depends on the same parameters as the model presented here.

Design parameters of the fibre-reinforced multilayer composite

The extensional stiffness matrix **D**, defined by equation (8), depends on the mechanical properties of the reinforcing fibres and the matrix, the fibre density and orientation in each particular layer of the composite, as well as the number of layers and their thickness. Thus, the components of this matrix can be written in the form:

$$D_{ij} = D_{ij}(E_a^k, \nu_a^k, E_m^k, \nu_m^k, \rho_a^k, \theta_k, n, h_k) \tag{9}$$

and they are expressed in terms of the design parameters introduced for the fibre-reinforced multilayer composite. Each of these parameters influences the mechanical properties of a composite material. However, the full advantages of such materials are obtained when the fibres are optimally layouted and oriented in each layer with respect to the assumed objective behavioural measure in the optimisation process under the structure's actual loading conditions. Therefore, in this paper's analysis, an angle of fibre orientation θ_i^k at any point of each k-th layer is selected as the design parameter which should be determined in the optimisation process.

The angle of fibre orientation θ_i^k at any point of the k-th layer defines the shape of the reinforcing fibres in the composite material (Figure 2). These design parameters can be constant in each layer (the fibres are rectilinearly spaced in the matrix), or can vary through the layer domain (the fibres are placed curvilinearly in the matrix). In this second case, the fibre orientation θ_i^k is treated as an angle between the tangent to the fibre and x-axis at any point of the k-th layer, and can vary within the layer domain. To describe the shape of reinforcing fibres a polynomial, Bezier or B-spline representation is introduced in the present paper, and the parameters defining this particular representation can be treated as design parameters.

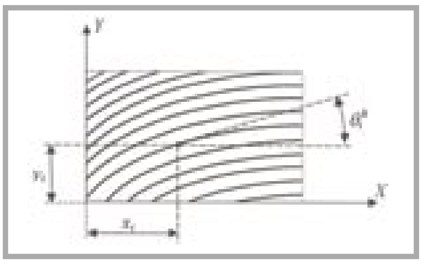

__Figure 2.__ Fibre orientation at any point of a layer.

Mean Stiffness Design of Composite Structure

The problem of optimal design of fibre shape and orientation in a multilayer composite will be dealt with while assuming the maximum mean stiffness of composite structure subjected to service loading.

Let us now consider a thin two-dimensional structure made of the composite material (Figure 3). This structure has a uniform thickness t; it is supported on the boundary portion S_U and loaded by traction $\mathbf{T} = [T_x, T_y]^T$ acting along the boundary portion S_T.

Under the applied loading, the structure undergoes some deformations described by displacement field $\mathbf{u} = [u_x, u_y]^T$.

The generalised plane stress

$$\sigma = [\sigma_x, \sigma_x, \tau_{xy}]^T$$

and strain

$$\varepsilon = [\varepsilon_x, \varepsilon_x, \gamma_{xy}]^T$$

fields, induced in the deformed structure, are interrelated by equation (7) with the extensional stiffness matrix **D** defined by equation (8).

Thus, the behaviour of structure is described by the equilibrium equation and kinematic equation:

$$div\, \sigma = 0 \quad \varepsilon = \left[\frac{\partial u_x}{\partial x}, \frac{\partial u_y}{\partial y}, \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)\right]^T \tag{10}$$

supplemented with the boundary conditions specified as follows:

$$\mathbf{u} = \begin{Bmatrix} u_x \\ u_y \end{Bmatrix} = 0 \quad \text{on } S_U$$
$$\mathbf{T} = \sigma\,\mathbf{n} \Rightarrow \begin{Bmatrix} T_x \\ T_y \end{Bmatrix} = \begin{Bmatrix} \sigma_x n_x + \tau_{xy} n_y \\ \tau_{xy} n_x + \sigma_y n_y \end{Bmatrix} \text{ on } S_T \tag{11}$$

where **n** denotes the unit-normal vector on the external boundary S.

The mean stiffness design is developed by using energy approaches [8]. The total potential energy Π_U of the structure is defined as the sum of total strain energy and work done by external load, and can be written in the form:

$$\Pi_U = \frac{1}{2}\int_A \varepsilon^T D \varepsilon t dA - \int_{S_T} u^T T dS_T \rightarrow max \tag{12}$$

Using the boundary conditions appearing in (11) and equation (10), we can write the following equality:

$$\int_{S_T} \mathbf{u}^T \mathbf{T} dS_T = \int_V \sigma^T \varepsilon dV = \int_A \varepsilon^T \mathbf{D} \varepsilon t dA \tag{13}$$

Finally, using equality (13) in equation (12) we obtain:

$$\Pi_U = -\frac{1}{2}\int_{S_T} \mathbf{u}^T \mathbf{T} dS_T \rightarrow max \tag{14}$$

and then the problem of mean stiffness design for a composite structure can be formulated as follows:

$$max(\Pi_U) \quad \text{or} \quad min\left(\int_{S_T} \mathbf{u}^T \mathbf{T} dS_T\right) \tag{15}$$

with respect to the shape or orientation parameters of the reinforcing fibres in its particular layers.

Optimisation of the Composite Structure using the Genetic Algorithm

The optimisation problem discussed in previous sections will be performed with the aid of the genetic algorithm. It is different from traditional optimisation techniques used in engineering design problems. The main idea behind this approach is to use the power of evolution to solve the optimisation problem.

A brief introduction to the genetic algorithm method is discussed in this section. First, the problem is re-defined in order to allow the use of the genetic optimisation. The steps of the genetic algorithm are outlined by presenting all its operators.

Problem formulation

The optimal design of the shape and orientation of reinforcing fibres in a mul-

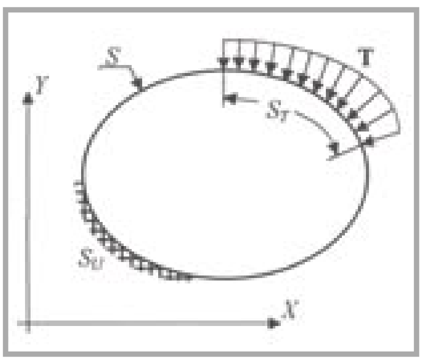

__Figure 3.__ Two-dimensional composite structure subjected to service loading.

tilayer composite is considered, so that structure made of this material should be as stiff as possible. Thus, the optimisation problem can be written in the following form:

$$STIFF_{max}(\theta) = min\left(\int_{S_T} \mathbf{u}^{\mathrm{T}}\mathbf{T}dS_{\mathrm{T}} \right) \equiv max(\Pi_{\mathrm{U}}) \qquad (16)$$

The potential energy Π_U is selected as the objective function in this process, since the genetic algorithms only solve the maximisation problem. Thus, the minimisation of the work potential is replaced by the maximisation of potential energy. The objective function Π_U is expressed by (12), where the stiffness matrix **D** has the form (9). The maximum mean stiffness of a composite material is considered with respect to the set of the variable vector $\theta = [\![\theta_1, \theta_2, \ldots, \theta_m]\!]$, whose components are angles of fibre orientation or the parameters defining these angles. Each variable θ_i^k defines the layout of the so-called directional fibre at any point in the k-th layer of a composite structure, and it is treated as a design parameter. Their optimal value will be derived during the optimisation procedure.

Basis of the genetic algorithm

The simple genetic algorithm will be applied in the optimisation process. Its block diagram is shown in Figure 4. The detailed description of such an algorithm is given, for instance, in [9,10].

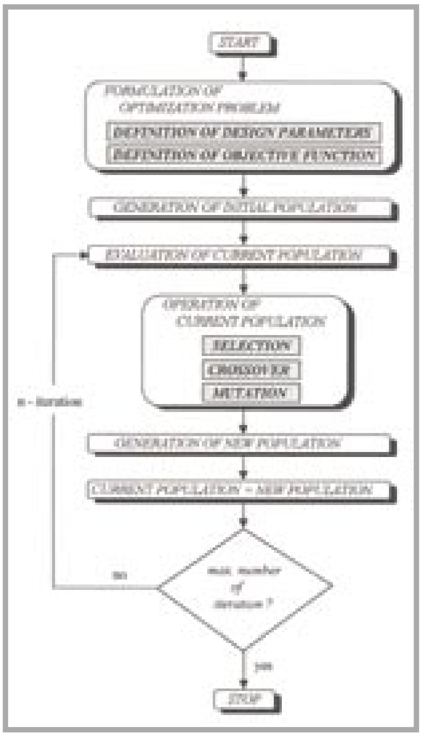

Figure 4. *Block diagram of the simple genetic algorithm.*

At first, in order to solve the optimisation problem formulated above, each design parameter θ_i^k is coded in a binary string. For instance, for the m independent angles of fibre orientation defining the layout of reinforcing fibres in all layers of a composite structure, the variable vector $\theta = [\![\theta_1, \theta_2, \ldots, \theta_m]\!]$ is represented as follows:

$$\underbrace{1001...01}_{\theta_1} ... \underbrace{0101...00}_{\theta_2} ... \underbrace{0010...10}_{\theta_\mu} \qquad (17)$$

Each variable θ_i is coded in l_i bits. The length of the string l_i for the i-th angle of fibre orientation can be determined by the following relationship:

$$2^{l_i} \geq \left(\theta_{i(max)} - \theta_{i(min)}\right) 10^{dok} \qquad (18)$$

where l_i is the smallest integer representing the solution of this inequality. The notation *dok* is used for the desired accuracy of the solution to the given problem, while $\theta_{i(min)}$ and $\theta_{i(max)}$ denote the variable bounds for the i-th design parameter θ_i.

Thus, a complete string (also referred to as a chromosome), which represents all design parameters, has a length l and is given by:

$$l = \sum_{i=1}^{m} l_i \qquad (19)$$

where m is the number of design parameters in the optimisation problem under consideration.

An initial population of N chromosomes is created in order to start the genetic simulation. Each chromosome in this population represents a point in space of the design parameters, and describes a possible solution to the given problem. All strings are created randomly, which thus guarantees a very great population diversity.

Thereafter, all chromosomes in the current population are evaluated by means of the values of the objective function. Each string is decoded using the equation (20), and then the potential energy Π_U for the decoded values of fibre orientation θ_i is calculated in this step of the genetic algorithm:

$$\bigwedge_{j=1..N} v(ch_j) = \Pi_U(\theta_1, \theta_2, \ldots, \theta_m) \qquad (21)$$

Thus, a fitness value $v(ch_j)$ is assigned to each chromosome in population. This value is related to the value of an objective function for the decoded string of design parameters.

The population is operated by three main operators of the genetic algorithms. They are selection, crossover and mutation.

Selection is mainly responsible for the search aspect of genetic algorithms. The essential idea in this operator is that 'good' strings are picked from the current population and multiple copies of them are created. As a result of this process, 'bad' strings are eliminated from the population and do not undergo any further consideration.

Each chromosome in the current population is selected with a probability proportional to the chromosome's fitness value $v(ch_j)$. The probability p_j for selecting the j-th chromosome to the new population is given by:

$$p_j = \frac{v(ch_j)}{\sum_{j=1}^{N} v(ch_j)} \qquad (22)$$

The crossover operator randomly recombines chosen two chromosomes by exchanging some portion of the strings between them, as shown for instance in the following scheme:

$$\begin{array}{c|c} 00 & 000 \\ \mathbf{11} & \mathbf{111} \end{array} \Rightarrow \begin{array}{c|c} 00 & \mathbf{111} \\ \mathbf{11} & 000 \end{array} \qquad (23)$$

The crossover point is performed randomly. It is intuitive from this approach that good chromosomes from either parent are combined to form a better child chromosome. However, in order to preserve some good strings, not all strings in the current population are used in the crossover. This operation is carried out with a crossover probability p_c. Besides, only $(1\text{-}p_c)100\%$ of the population are put into the new population.

Mutation introduces random modifications to create a better chromosome in the population.

$$00\mathbf{1}11 \Rightarrow 00\mathbf{1}0\mathbf{1} \qquad (24)$$

As shown in equation (23), this operator alters a string locally and changes a 1 to a 0 or vice versa with a very small mutation probability p_m. Mutation is necessary to maintain diversity in the population.

$$\theta_\iota = \theta_{i(min)} + decimal\ \underbrace{(1001...01)}_{\theta_i} \left(\frac{\theta_{i(max)} - \theta_{i(min)}}{2^{l_i} - 1} \right)$$

Equation 20.

Finally, the new population of solutions is created; then, the single cycle of the genetic algorithm, which is known as a generation in genetics terminology, draws to an end. This new population is again operated by the above three operators and evaluated. Each successive generation contains better 'partial solutions' than previous generations, and converges towards the global (or near global) optimum. This procedure is continued until the termination criterion is satisfied. Usually, a genetic simulation is terminated when a specified number of generations have elapsed.

Illustrative Example

A simple example will be discussed in this section in order to show the applicability of genetic simulation for optimal shape design of reinforcing fibres in a composite structure. Let us consider the structural component shown in Figure 5.

This structural component is made of a 3-layer composite material. Each layer of the composite consists of epoxy matrix (E_m=3·10^3 [MPa], ν_m=0.4) reinforced with a ply of glass fibres (E_w=7·10^4 [MPa], ν_w=0.25) and has the thickness h_k=0.005 [m]. The fibres are regularly spaced in the matrix, with a constant density ρ_w=0.5 in each particular layer.

The problem deals with the optimal layout of reinforcing fibres in the multilayer composite in the case of the mean stiffness design of the structure made of this material. This design problem was explicitly formulated in Section 4.1, and will be discussed here for three classes of fibre shape.

In the first case, each layer in the composite material of the structural component will be reinforced with one family of straight fibres. Thus, the angle of fibre orientation θ_1 in the middle layer and the angle of fibre orientation θ_2 in the outer layers are treated as design parameters which should be determined in the genetic optimisation process (see the section 'Basis of the genetic algorithm').

This process was carried out with the following parameters:
- the lower and upper bounds for design parameters θ_1 and θ_2 are 0° and 180°, respectively;
- the population size N=100;
- the crossover probability p_c=0.9;
- the mutation probability p_m=0.005;
- the number of generations n=300.

The objective function Π_U for the decoded values of fibre orientation θ_i was calculated using the finite element method [11] in the analysis step of the population evaluation. In this approach to the analysis of the mechanical system, the considered structural component defining a continuum is discretised into 48 four-node quadrilateral elements. The history of the genetic simulation is presented in Figure 6.

We can observe from this figure that the genetic algorithm reached the optimal solution after 104 generations. Thus, the calculated optimal values of design parameters θ_1 and θ_2 are 140.78° and 110.09° respectively with respect to the x-axis (θ=0°). This design corresponds to the maximum stiffness of the structural component in the assumed class of shape for reinforcing fibres. The fibre layout in the particular layers of the composite material after the optimisation process with one family of straight fibres is shown in Figure 7.

This case was next verified using the gradient-oriented search method; the plot of work potential W_P versus the angles of fibre orientation θ_1 and θ_2 in particular layers is presented in Figure 8.

We can observe from Figure 8 that the maximum stiffness for the structural component made of the composite material is obtained when the angles of orientation for straight reinforcing fibres are θ_1=141° in the middle layer and θ_2=110° in the outer layers. In addition, it is easy to verify that these optimal values correspond to the increase in the construction stiffness by 75.3% in comparison to the design with fibre orientation θ_1=45° and θ_2=45° respectively in the particular layers, for which this structural component will be the most deformable.

The case of the reinforcing fibres constituting an open polygon is considered next. For this case, the fibre layout is defined

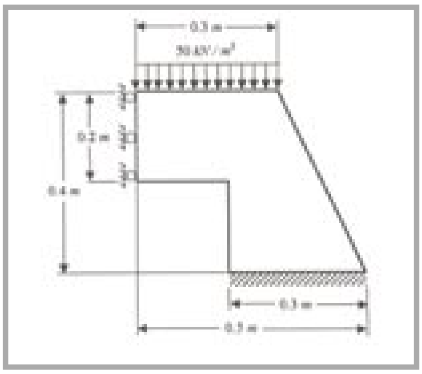

Figure 5. Structural component subjected to the load and boundary conditions.

by 5 independent design parameters: the angles of orientation θ_1, θ_2, θ_3 in the middle layer (three families of straight fibres), and θ_4, θ_5 in the outer layers (two families of straight fibres).

The optimal values of the design parameters after the optimisation process finishes are θ_1=103.50°, θ_2=114.49°, θ_3=170.77°, θ_4=110.18° and θ_5=122.58° (Figure 8). This optimal design, as in the first case, corresponds to an increase in structure stiffness of 78%.

Finally, the composite material of the construction will be reinforced with a family of parabolic fibres. Thus, the shape of the reinforcing fibres is assumed in the form $y=a_1(x-p_1)$ in the middle layer, and $x=a_2(y-p_2)$ in the outer layers, where the coefficients a_1, p_1, a_2, p_2 are treated as design parameters.

In this case, the calculated optimal values of the shape design parameters are as follows: a_1=-3.31, p_1=0.07 and a_2=-0.87, p_2=0.03. This design corresponds to the maximum stiffness of the structural component in the assumed class of fibre shape. The optimal structural component with the family of parabolic fibres is shown in Figure 9.

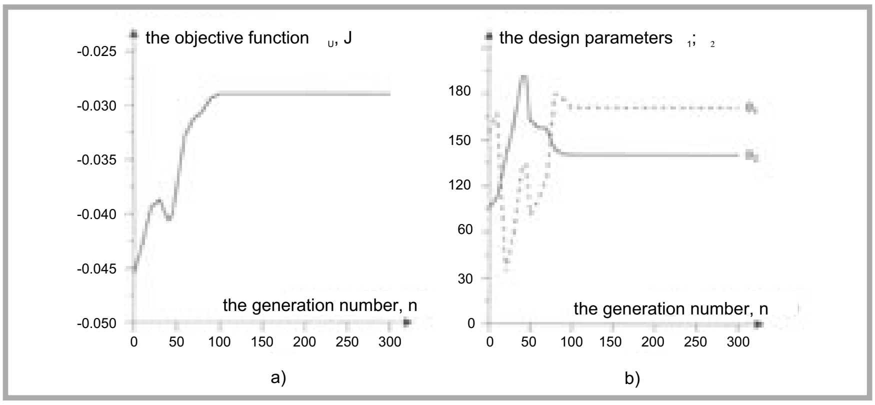

Figure 6. History of optimisation using the genetic algorithm; a) - dependence of objective function vs. generation number, b) - dependence of design parameters vs. generation number.

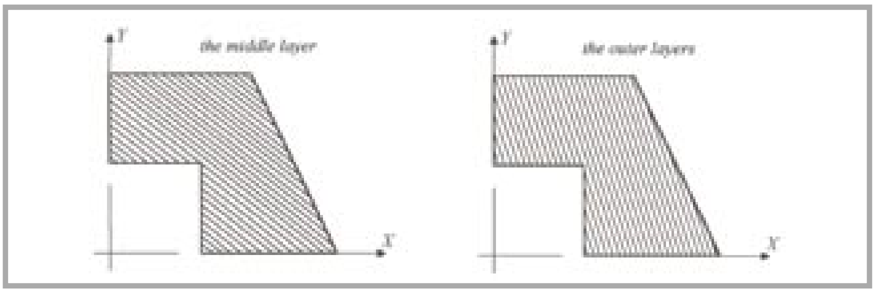

Figure 7. Optimal structural component with one family of straight fibres.

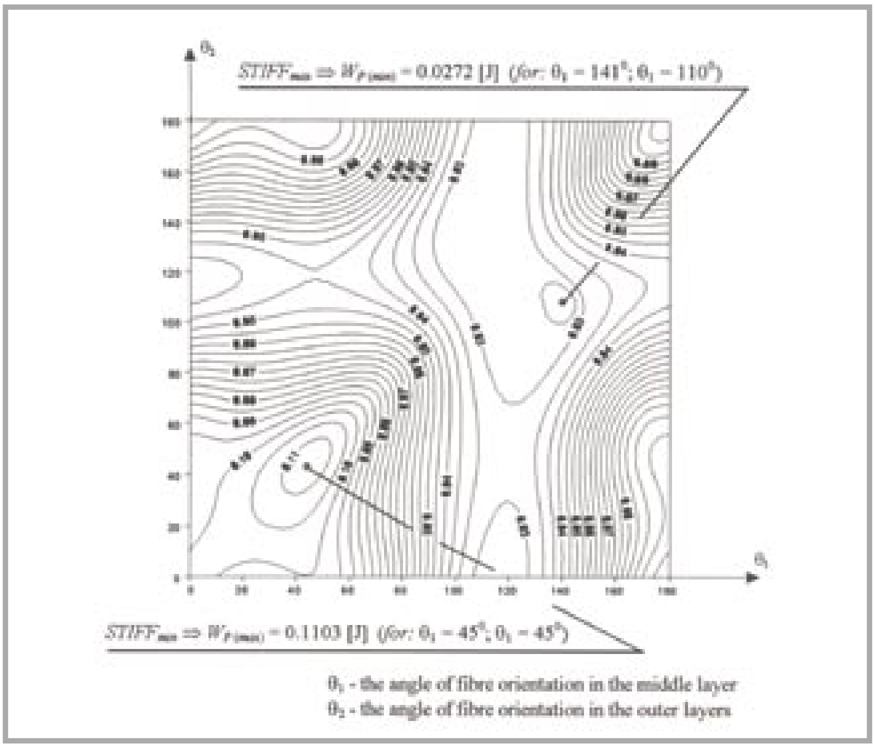

Figure 8. Mean stiffness of structure versus different angles orientation.

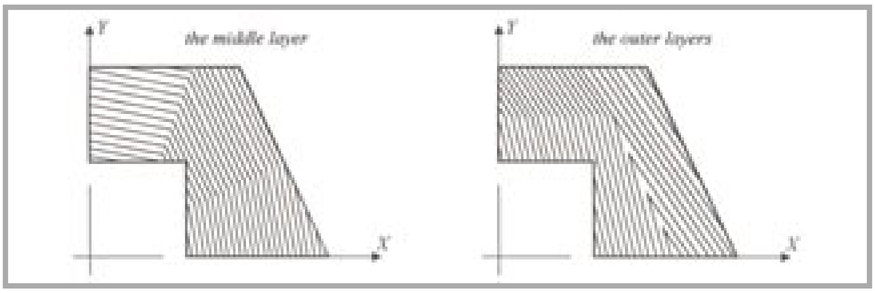

Figure 9. Optimal structural component with families of straight fibres.

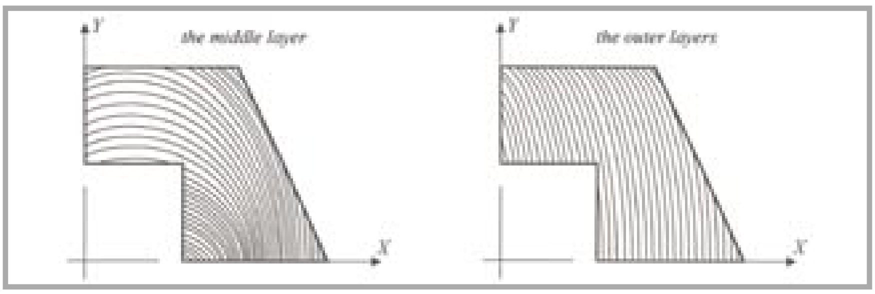

Figure 10. Optimal structural component with a family of parabolic fibres.

Conclusions

The results presented allow us to state that the full advantages of composite materials are obtained when the fibres are optimally layouted and oriented in each layer with respect to the assumed objective behavioural measure in the optimisation process under the actual loading conditions of the structure. Furthermore, the genetic method used in the design process of a structure is a simple and very effective way of quickly finding a reasonable solution to a given problem. Thus, it can serve as an alternative technique to the classic methods applied in the question of designing composite materials.

This results are a starting point for the optimal design of a real structure made of a composite material which works under actual loading conditions. An application to the more complicated design problems of composite structures will be presented in subsequent papers.

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