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## Introduction

The Kinemat 200 textile machine was created at the Technical University of Sofia by a team managed by Professor Milkov [1]. The Kinemat 200 was designed to produce non-woven floor coverings. The face layer of the coverings consists of bands, fibres or textile cables. The average productivity of the machine is $2 \mathrm{~m}^{2}$ per minute.

A simplified drawing of the loop-forming device of the Kinemat 200 is presented in Figure 1. The loop forming mechanisms (1) and (2) move the pushing ruler (3) along a prescribed closed trajectory, so that it penetrates cyclically into the grooves of the loop-forming drum (4). During these penetrations, the textile material is pushed into a groove and one textile loop is for-

# Synthesis of Loop-Forming Mechanisms of the Kinemat 200 Machine for Non-Woven Coverings 


#### Abstract

The paper presented concerns the design of the kinematics of the loop-forming mechanisms of a machine for non-woven coverings. Two eight-link mechanisms are optimised by means of the mini-max Chebyshev theory. The Chebyshev Theorem is applied on the assumption that the continuous objective functions possess the same features as the generalised Chebyshev polynomials if the numbers of the maximum roots of both are equal. In addition, some basic principles that have been used during the designing process of the textile machine are presented.


Key words: non-woven covering, loop-forming mechanism, mini-max design, kinematic synthesis, Chebyshev's Theory.
med. A constant velocity ratio transmission consisting of a teeth-chain reducer (5), wave reducer (6) and gear train (7) connects the loop-forming drum and the input link of the loop-forming mechanisms, and provides uniform rotation by the electrical motor (8). The flywheel (9) connects the two loop forming mechanisms.

The loop-forming mechanisms are the most complicated and the most important devices in the Kinemat 200, because of its various technological functions and the significant influence on the quality of the non-woven coverings produced.

This paper describes the synthesis methods and the obtained solutions which were used as a basis for designing the loop-forming devices of the Kinemat 200.

## Formulation of the Synthesis Problem

The first part of the synthesis task concerns the mechanism structure. The connectivity of the mechanisms follows from
the basic constructive principles, which have been adopted during the process of the machine building. The first of these principles is the constructive independency of the mechanisms. This independency means that every mechanism has to be designed as a unique module, so it can be used and tested independently. The second principle is continuous and uniform rotation of the transport link, which is the loopforming drum. This link is designed with a large radius and a large length. Thus, the uniform rotation eliminates the inertia forces of the link with the biggest mass. The third principle concerns the possibilities of the loop-forming mechanism to provide independent adjustments of the transport and working motions: the transport motion allows the pushing ruler to follow the rotating grooves with minimum error, and the working motion provides the necessary penetration of the pushing ruler into the drum grooves.

Two mechanisms structures were chosen (Figure $2 \mathrm{IM}, \mathrm{RM}$ ) according to the requirements mentioned above. The so-called


Figure 1. Loop-forming device of Kinemat 200; 1, 2 - loop-forming mechanisms, 3 -pushing ruler, 4 -loop-forming drum, 5-teeth-chain reducer, 6 -wave reducer, 7 - gear train, 8 - electrical motor, 9 -flywheel.


Figure 2. Ideal (IM) and real (RM) loop-forming mechanism; 1,6,7- slider-crank mechanism; 1,2,3 - crank-shaper mechanism; 3,4,5 - tangent mechanism (IM) or cosine mechanism (RM); 7 - slider; 8 - loop-forming drum.
ideal mechanism (IM) consists of a slidercrank mechanism (links 1, 6 and 7), and a crank-shaper mechanism (links 1, 2 and 3). The offset of the slider-crank mechanism is variable because of the vertical translation of link 5, which accomplishes the transport motion by means of the tangent mechanism consisting of links 3,4 and 5. The pushing ruler is held fixed on the slider (7). The radius of the loop-forming drum (8) is selected to be as large as the motion of the grooves in which the ruler penetrates could be considered as a vertical straight-line translation. Thus it is assumed that the motion of the loop-forming grooves is nearly translation. The working motion depends mainly on the dimensions $\mathrm{OD}=\mathrm{R}$ and $\mathrm{DE}=1$ because of the very weak influence of the variable offset $y_{E}$. The formulae presented as equations (1) and (2) describes the transport and the working motion:

$$
\begin{equation*}
y_{E}=b \frac{\sin \varphi}{\mu+\cos \varphi} \tag{1}
\end{equation*}
$$

$$
\begin{equation*}
x_{E}=R\left[\cos \varphi+\sqrt{\frac{1}{\lambda^{2}}-\left(\sin \varphi+\frac{y_{E}}{l}\right)^{2}}\right] \tag{2}
\end{equation*}
$$



Figure 3. Minimised aim function of IM; $\Delta y_{E}=f(\varphi)$
where:

$$
\begin{gathered}
b=x_{C}-x_{B}, \quad \mu=d / r, \quad d=O B=x_{B}, \\
R=O D, \quad l=D E, \quad \text { and } \lambda=R / l .
\end{gathered}
$$

The angle $\varphi$ notes the input link 1 orientation as shown in Figure 2.

The so-called real mechanism (RM) is compounded with a similar structure to the ideal one (Figure 2), but the tangent mechanism that consists of links 3,4 and 5 is changed with a cosine mechanism. The position functions in this case are these desrcibed by equations (3) and (4).

$$
\begin{gather*}
y_{M}=c \frac{\sin \varphi}{\sqrt{1+\mu^{2}+2 \mu \cos \varphi}}  \tag{3}\\
x_{M}=R\left[\cos \varphi+\sqrt{\frac{1}{\lambda}-\left(\sin \varphi-\frac{y_{M}}{l}\right)^{2}}\right] \tag{4}
\end{gather*}
$$

Here $c=B C$, and the rest of the notations are the same as in formulae (1) and (2).

According to the information given above, the synthesis task can be formulated in two conditions. The first one concerns the working motion, and it means that for an interval of the input angle $\Delta \varphi=\varphi_{b}-\varphi_{a}$ (called the working stroke interval), the output link 7 has to penetrate into the groove to a given depth $h$ and to leave it at the end of the interval $\varphi_{b}$. The beginning of the working interval is noted as $\varphi_{a}$. This condition can be fulfilled when the inequality
$\Delta x=x\left(\varphi_{m}\right)-x\left(\varphi_{a}\right)=x\left(\varphi_{m}\right)-x\left(\varphi_{b}\right) \geq h$
is true. The input angle $\varphi_{m}$ corresponds to the maximum value of the functions (1)
or (3). The transport condition can be fulfilled if the target function

$$
\begin{equation*}
\Delta y=y_{d}-y \tag{6}
\end{equation*}
$$

is minimised, where

$$
\begin{equation*}
y_{d}=\frac{p}{2 \pi} \varphi \tag{7}
\end{equation*}
$$

is the position of the transport link and $y=y_{E}$ for IM or $y=y_{M}$ for RM determined by formulae (2) or (4) accordingly.

## Solution of the Synthesis Problems

The equations (1) and (2), respectively (3) and (4) show that the selected structures of the loop-forming mechanisms allow the synthesis task to be solved separately, at first for the working condition and secondly for the transport condition.

As the limits of the working stroke period are chosen according to the inequalities

$$
\begin{equation*}
-\pi / 2<\varphi_{a} \leq-\pi / 3 \tag{8}
\end{equation*}
$$

$$
\begin{equation*}
\pi / 2>\varphi_{a} \geq \pi / 3 \tag{9}
\end{equation*}
$$

it is easy to prove that for both mechanisms the inequality (5) is roughly ful filled if the dimension of the crank is $R>(1.1-1.3) h$.

The transport conditions for both mechanisms are approximately satisfied by the use of Chebyshev's Best Approximation Theory. After exploring the second derivation of the expression (1) with respect to $\varphi$ :

$$
\begin{equation*}
y^{\prime \prime}=\frac{d^{2} y_{e}}{d \varphi^{2}}=-\frac{b\left(\mu^{2}-\mu \cos \varphi-2\right) \sin \varphi}{(\mu-\cos \varphi)^{3}} \tag{10}
\end{equation*}
$$

and proving that the aim function

$$
\begin{equation*}
\Delta y_{E}=y_{d}-y_{E} \tag{11}
\end{equation*}
$$

has no more than 2 roots in the working stroke interval, it is assumed that the objective function (11) can be presented as a first order Chebyshev polynomial [3]. The function (11) has a zero root and is symmetrical with respect to this root. This is the reason for considering only half of the approximating interval. In this half interval for $\varphi \in\left(0, \varphi_{b}\right]$ the function (11) has to reach its maximum limit deviations in 3 points. For the whole working stroke interval, it follows that the number of the maximum deviations is 6 . On applying the Chebyshev theorem, the system presented in the equations (12) is obtained:

$$
\begin{align*}
& \Delta y_{E}\left(\varphi_{1}\right)=\frac{p}{2 \pi} \varphi_{1}-b \frac{\sin \varphi_{1}}{\mu+\cos \varphi_{1}}=L  \tag{14}\\
& \Delta y_{E}\left(\varphi_{2}\right)=\frac{p}{2 \pi} \varphi_{2}-b \frac{\sin \varphi_{2}}{\mu+\cos \varphi_{2}}=-L \\
& \Delta y_{E}\left(\varphi_{2}\right)=\frac{p}{2 \pi} \varphi_{b}-b \frac{\sin \varphi_{b}}{\mu+\cos \varphi_{b}}=L \\
& \Delta y_{E}^{\prime}\left(\varphi_{1}\right)=\frac{p}{2 \pi}-b \frac{1+\mu \cos \varphi_{1}}{\left(\mu+\cos \varphi_{1}\right)^{2}}=0  \tag{15}\\
& \Delta y_{E}^{\prime}\left(\varphi_{2}\right)=\frac{p}{2 \pi}-b \frac{1+\mu \cos \varphi_{2}}{\left(\mu+\cos \varphi_{2}\right)^{2}}=0
\end{align*}
$$

## Equations 12

The unknowns of the system (12) are $b$, $\mu$, the maximum deviation $L$ and the input angle positions $\varphi_{1}$ and $\varphi_{2}$ in which the aim function reaches its maximum deviations. The third maximum deviation point is the end of the working stroke interval $\varphi_{b}$. The system (12) is solved numerically for $p=4.23 \mathrm{~mm}$ and $\varphi_{b}=89^{\circ}$, so it is found that $L=0.0027922 \mathrm{~mm}$, $b=1.8013, \mu=1.7094, \varphi_{l}=0.51046578$ and $\varphi_{2}=1.2845$. The minimised aim function (11) is shown in Figure 3.

The transport condition of RM can be approximately satisfied if the aim function

$$
\begin{equation*}
\Delta y_{M}=y_{d}-y_{M} \tag{13}
\end{equation*}
$$

is minimised in the working stroke interval. The second derivative of the function (13) with respect to the angle $\varphi$ pre-


Figure 4. Space curve of the optimal parameters of $R M$.


Figure 5. Minimised aim function of RM; $\Delta y_{M}=f(\varphi)$

$$
y_{M}^{\prime \prime}=\frac{d^{2} y_{M}}{d \varphi^{2}}=\frac{c \sin \varphi}{\left(1+\mu^{2}+2 \mu \cos \varphi\right)^{5 / 2}}\left(1-\mu^{2}+\mu \cos \varphi+\mu^{4}+\mu^{3} \cos \varphi+\mu^{2} \cos \varphi\right)
$$

$$
\begin{aligned}
& \Delta y_{M}\left(\varphi_{1}\right)=\frac{p \varphi_{1}}{2 \pi}-\frac{c \sin \varphi_{1}}{\sqrt{1+\mu^{2}+2 \mu \cos \varphi_{1}}}=-L \\
& \Delta y_{M}\left(\varphi_{b}\right)=\frac{p \varphi_{b}}{2 \pi}-\frac{c \sin \varphi_{b}}{\sqrt{1+\mu^{2}+2 \mu \cos \varphi_{b}}}=L \\
& \Delta^{\prime} y_{M}\left(\varphi_{1}\right)=\frac{p}{2 \pi}-\frac{c\left(1-\mu^{2}\right) \cos \varphi-c \mu \sin ^{2} \varphi_{1}}{\sqrt{\left(1+\mu^{2}+2 \mu \cos \varphi_{1}\right)^{3}}}=0 .
\end{aligned}
$$

## Equations 14 and 15

sented in equation (14) has only complex roots in the half-approximation interval except for $\varphi=0$. There is the same symmetry with respect to the coordinate system origin. These features prove that for the half interval, which does not include a zero point, the aim function has no more than 1 root. As the 0 -order Chebyshev polynomial possesses the same property, it is assumed that the function (13) is the same order polynomial. It means that for the half-approximation interval there are two points in which the function reaches its maximum deviation. According to the Chebyshev theorem, the system presented in (15) proceeds.

The number of the unknowns ( $c, \mu, L$ and $\varphi_{1}$ ) here is greater than the equation system number. This means that only 3 parameters can be calculated by the system (15). The optimisation method used here involves two steps. The first concerns varying the dimension $c$ within a given interval $1.0 \leq c \leq 2.8 \mathrm{~mm}$ with constant increment. The second step is the solution of the system (15) with respect to $\mu, L$ and $\varphi_{1}$. The minimum value of the maximum deviation $L$ gives the optimal values of the remaining unknown parameters. The space curve shown in Figure 4 describes the dependency of maximum deviation $L$ from $c$ and $\mu$. In this way, the following optimal parameters for $c=1.468, L=0.0273273$ $\mathrm{mm}, \varphi_{1}=0.771$, and $\mu=1.020543$ are obtained. The graph of the minimised aim function of RM is shown in Figure 5.

The remaining constructive parameters of both mechanisms are selected or calculated according to the necessary strength of the links and its dynamical properties.

## Conclusions

- The proposed synthesis methods provide opportunities to find the best ap-
proach of the aim functions under consideration. This means that the maximum deviations obtained are the least possible within the space of the optimised parameters.
- Although the minimised aim function of the IM has two roots more than the similar function of the RM, and the error of the IM is 10 times larger, both mechanisms are completely suitable for the non-woven covering production.
- Common disadvantages of both loopforming mechanisms are the large number of links and the presence of slider pairs. These disadvantages are avoided in the next generation loop-forming devices of the Kinemat 200 machine. In these mechanisms, the number of the links is decreased and only rotate pairs are used [4-6].


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