Theoretical Profile of Ring-Spun Slub Yarn and its Experimental Validation

Abstract
A mathematical model for the yarn count of ring-spun slub yarn was established to predict the yarn profile based on process parameters such as the fibre length, the velocities of rollers and the time of over-feeding. The theoretical study shows that the slub length depends mostly on the process parameters above. The actual slub length is a fibre length greater than the slub length designed, and the actual length of basic yarn is a fibre length less than the length of basic yarn designed. The validity of the model was then verified using four sets of experiments. The experimental results agreed well with the model predictions and showed that the present model had high prediction accuracy, which may aid in the design and production of a desired slub yarn with controlling spinning parameters.

Key words: slub yarn, mathematical model, slub profile, slub length, basic yarn length.

■ Introduction
Fancy yarns, because of their abundance of special aesthetic and structural effects of the textile materials in which they are used, have attracted much interest in recent years [1 - 3]. These yarns give decorativeness, improve the appearance of a garment and change the end-use properties of a fabric [4 - 6].

Slub yarn, as one of the fancy yarns, is now increasingly used for almost all kinds of textile products, such as denim garments, shirts and upholstery, due to its unique style and bamboo-like profile [7 - 9]. To date, a variety of methods are available for producing slub yarns, like ring-spun slub yarns and rotor-spun slub yarns. In these methods, ring spinning is the earliest and also the most common processing technology in slub yarn production at the present time. A well-known method in this process is to modify the ring spinning frame in such a way that the intermittent acceleration of the drafting rollers causes constantly varying degrees of draft to be applied [10].

In earlier research on slub yarn, Testore and Minero [11] proposed and discussed a classification and standardisation for the determination of the fundamental parameters that characterise sub-type fancy yarns. Grabowska [12] studied and characterised the basic parameters describing the structure and tensile properties of slub fancy yarn produced on a ring-twisting frame. Wang and Huang [13] discussed and analysed the parameters of rotor spun slub yarns in detail. Recently Lu, Gao and Wang [14] established a mathematical model based on the bar torsion model to describe twist distribution in slub yarn. Liu et al. [15] presented an analytical method of determining slub yarn geometrical parameters based on a 2D visualisation image of a slub yarn. However, there is still a lack of adequate understanding of this process, especially the relationship between process parameters and the profile of slub yarn in the spinning process. For example, the actual slub length of slub yarn is not exactly equal to the length designed in actual yarn production.

In this work, the intermittent acceleration of the drafting rollers by controlling stepper motors in ring spinning, causing constantly varying degrees of draft to form randomly distributed slubs (thick places) in the yarn, was adopted to experimentally and theoretically study the relationship between the yarn count and some important process parameters such as the fibre length, roller velocity, running time.

■ Mathematical model
In this study, slub yarn was produced by changing the speeds of the back-mid rollers while the front roller ran with a constant speed to deliver the sliver at a constant rate, which caused a variation in the resultant yarn fineness. Slub yarns were considered using the overfeeding of the back-mid roller method in the 3-over-3 apron drafting system. After initial stretching and drafting between the back rollers and middle rollers, the sliver was fed into the drafting zone (middle and front rollers). The speed of back-mid rollers varied with an intermittent acceleration, which was equivalent to additional fibre feeding into the drafting zone, allowing the production of slub yarns.

Generally the characteristics of slub yarn depend mainly upon how and where the fibres in the sliver are accelerated and overfed, hence the speed, acceleration...
time, deceleration time and highest speed
time of the accelerated rollers are of cru-
cial importance.

In order to simplify the problem, we can
make some assumptions:

a) The drawing process is ideal for slab
yarn production, *i.e.*, a part of fibres,
whose headend, which is defined
as the leading end of the fibre while
moving, arrive at the front roller nip
in a ring frame, are then gripped
and move immediately at a linear speed
of the front roller, while any other fibres
in the drawing zone keep their original
speed of back-mid rollers. There are
no floating fibres during drafting.

b) All the fibres have the same fibre
length (*l*) and fibre fineness (*N_p*).

(c) The fineness of the sliver (**N_x**) and
distribution density of fibre headends
in the sliver (**m_x**) are constant. Thus the
number of fibres (**n_x**) in a cross-section
of sliver and the distribution den-
sity (**m_x**) of fibre headends in a sliver
can be expressed as

$$n_x = \frac{N_x}{N_p}$$

$$m_x = \frac{n_x}{l} = \frac{N_x}{N_p \times l}$$

If we consider the front roller nip as the
ordinate origin and take the traveling di-
rection of the yarn as the X axis towards
the right as positive, the distribution den-
sity (**m_x**) along the X axis can
be expressed as

$$m_x = \frac{m_x \times V_x(x)}{V_x}$$

where **V_x** is the linear velocity of the
front roller, and **V_x(x)** is the instantaneous
velocity of the back-mid roller when
the random position yarn moves through
the front roller nip.

The number of fibres in the cross-section
of yarn at a random position **x** can be expressed in the form of **Equation 4**.

$$n_x(x) = \int_{0}^{x} m_x \cdot V_x dx = \int_{0}^{x} \frac{m_x \times V_x}{V_x} dx = \frac{N_x}{N_p \times l \times V_x} \int_{0}^{x} V_x dx$$

where **n_x(x)** is the number of fibres at a
random position **x** in the X axis.

Thus the yarn fineness can be expressed as

$$N_x(x) = n_x(x) \times N_p = \frac{N_x}{l \times V_x} \int_{0}^{x} V_x dx$$

As described above, **N_x**, **l**, and **V_x** are con-
stant and we can let **N_x/lV_x** = **A**. Thus the
above equation can be simplified as

$$N_x(x) = A \int_{0}^{x} V_x dx$$

Thus the average linear density of slub
yarn between positions **x** and **x + Δx** can be written in the form of **Equation 7**,

$$\bar{N}_x = \frac{1}{\Delta x} \int_{0}^{\Delta x} N_x(x) dx = \frac{N_x}{\Delta x \times V_x} \int_{0}^{\Delta x} V_x dx^2$$

where the value of Δ**x** is very small.

During the formation of slub yarn, the
overfeeding of fibres, which can be
caused by an acceleration of the back-
mid rollers, which passes additional fibres
to the drawing zone, will generally
affect the size of the resulting slub.

Generally the more the overfeeding fi-
bres to the drawing zone, the greater the
slub. The size and profile of the slub will
also vary with a number of other vari-
ables, such as the linear velocities of
the drawing rollers, the acceleration time,
deceleration time and running time at
the highest speed of the back-mid rollers.

All of these may be varied to produce a
finished yarn with the qualities desired.

We assume that the linear velocity of
the back-mid rollers are **V_21** and **V_22**, res-
pectively, at the lowest and highest constant
speed during the running time of **t_1** and
**t_22** respectively. The acceleration time
of the back-mid rollers from the lowest
constant speed (**V_21**) to the highest con-
stant speed (**V_22**) is **t_21** and the deceleration
time from **V_22** to **V_21** is **t_22** as shown in
**Figure 1.a**. Here we define the whole
time of slub spinning as

$$t_2 = t_21 + t_22 + t_23$$

According to the spinning principle of
slub yarn, some basic conditions such as

$$t_1 V_1 \geq l, t_21 V_1 < l, t_23 V_1 < l, V_21 < V_22$$

and

$$V_22 < V_1$$

are almost always true.

$$V_x(x) = \begin{cases} V_1, & (0 \leq x < t_1) \\ V_2, & (t_1 \leq x < t_1 + t_21) \\ V_2 + V_22 - \frac{V_22 - V_1}{t_22} \left(x - t_1 - t_22\right), & (t_1 + t_21 \leq x < t_1 + t_22) \end{cases}$$

**Equations 8 and 9.**
Thus the velocities of the back-mid rollers can be written as Equation 8.

This function is a piecewise linear function with 4 pieces. In order to simply, we let

\[
\frac{V_{22} - V_{21}}{t_2 V_1} = B
\]

\[
\frac{V_{22} - V_{21}}{t_2 V_1} = C
\]

in the analysis below.

Theoretical prediction of the yarn number (linear density) and profile of ring-spun slub yarn

In the manufacturing process of slub yarn, the length \( t_{22} V_1 \) of yarn delivered from the front roller during the time of \( t_{22} \) plays a key role in the yarn number (linear density) and profile of the slub yarn. Generally the yarn number and profile of slub yarn mainly depends upon the relationship between the above length and that of fibres. Hence the yarn number (linear density) and theoretical profiles of slub yarn are analysed and discussed under the following conditions.

**Thickest yarn length of slub yarn greater than the fibre length, \( t_{22} V_1 > 1 \)**

In this condition, the thickest yarn length \( t_{22} V_1 \) of the yarn delivered from the front roller in the time of \( t_{22} \) is greater than the fibre length. The velocity of the back-mid rollers increases from the lowest speed \( V_{21} \) to the highest \( V_{22} \) during the time of \( t_{21} \), then keeps the speed during the time of \( t_{22} \), and finally decreases from \( V_{22} \) to \( V_{21} \). The graph of function \( V_{22}(x) \) can be drawn as the lower polyline in Figure 1.a according to Equation 8.

Substituting Equation 8 into Equation 6 and integrating, function \( N_{tx}(x) \) can be written as a piecewise continuous function with 8 pieces presented as Equation 9.

In the first piece, when \( 0 \leq x < t_1 V_1 - l \), we get \( N_{tx}(x) = A V_{22} x \). It is a linear function and the graph of the function is a straight line. In the second piece, it is a quadratic function and its shape is a parabola that opens up between the two ends,

\[
(t_1 V_1 - l, AV_{22} l) \quad \text{and} \quad ((t_1 + t_2) V_1 - l, AV_{22} l + AB(t_2 V_1)^2/2)
\]

The third piece is also a linear function, the range of which is

\[
[AV_{22} l + AB(t_2 V_1)^2/2, AV_{22} l + AB(t_2 V_1)^2/2]
\]

The fourth piece is also a quadratic function and its shape is a parabola that opens down. In the fifth piece, when

\[
(t_1 - t_2) V_1 - l < x < (t_1 + t_2 + t_22) V_1 - l,
\]

we obtain \( N_{tx}(x) = A V_{22} l \).

It is also a linear function, the graph of which being a straight line. The graphs of other pieces are similar and symmetrical to those of the first three pieces. According to the \( x \) value (domain) and graph of each piece, the piecewise graphs of all eight functions are drawn by taking each ‘piece’ and treating it as a separate function, as shown in Figure 1.a.

The yarn number (linear density), \( N_{tx}(x) \), indicates the yarn diameter or fineness to which that particular yarn has been spun. Hence the profile of slub yarn is similar to that of the function graph, as shown in Figure 1.b.

Additionally the three important parameters: the length of basic yarn \( L_b \), the slub length \( L_s \), and the length of the thickest yarn \( L_M \) can be obtained from Equation 9 and Figure 1.

\[
L_b = t_1 V_1 - l \quad (10)
\]

\[
L_s = t_2 V_1 + l \quad (11)
\]

\[
L_M = [(t_1 + t_2 + t_22) V_1 - l] + - [(t_1 + t_2) V_1] = t_22 V_1 - l \quad (12)
\]

From the above equations and discussion, we can draw the following conclusions:

1. Although the velocity of the back-mid rollers \( V_2 \) can be increased or decreased linearly, the linear density of slub yarn, \( N_{tx}(x) \), can not be increased or decreased linearly.

2. Although the velocity of the back-mid rollers \( V_2 \) has reached its maximum, the thickest part of slub yarn will not occur until after the yarn has run over the distance of the fibre length, or over a time of \( l/V_1 \).

3. In this condition, the actual length of the slub is \( t_2 V_1 + l \), i.e., the slub length is a fibre length longer than the slub length designed. The actual length of basic yarn is less than that of the basic yarn designed, i.e., \( t_1 V_1 - l \).

**Thickest yarn length of slub yarn less than the fibre length, \( t_{22} V_1 < l \)**

In this condition, the thickest yarn length \( t_{22} V_1 \) of the yarn delivered from the front roller during the time of \( t_{22} \) is less than the length of fibres. Generally the acceleration time \( t_{21} \) and deceleration time \( t_{22} \) of the back-mid roller are equal in order to produce high-quality slub yarn, i.e., \( t_{21} = t_{22} \). Thus we get \( B = C \). A graph of the function \( V_{22}(x) \) can be drawn as the lower polyline in Figure 2.a (see page 32) according to Equation 8.

Based on the function \( V_{22}(x) \), function \( N_{tx}(x) \) can be integrated as Equation 13 where \( N_{tx}(x) \) is a piecewise continuous function with 6 pieces. In this function, the first three pieces and sixth piece are the same as the first and eighth piece in
Equation 9. The fourth piece is a parabola that opens down with the range,
\[ AV_2l - AB(t_2V_1)^2/2, AV_2l \]

The fifth piece is a linear function, the range of which is
\[ AV_2l + AB(t_2V_1)^2/2, AV_2l - AB(t_2V_1)^2/2. \]

The graphs of all six functions are drawn in Figure 2.a. The profile of slub yarn is similar to that of the function graph, as shown in Figure 2.b.

Additionally the three important parameters: the length of the thickest yarn \( L_M \), the slub length \( L_s \) and yarn number (linear density) of the thickest yarn \( N_{tM} \) can be obtained from Equation 13 and Figure 2.a.

\[ L_M = 0 \quad (14) \]
\[ L_s = 2l + t_2V_1 \quad (15) \]
\[ N_{tM} = AV_2l \quad (16) \]

Two extreme conditions

The first condition \( t_2V_1 = 0 \)
In this condition, it is clear that \( t_2 = 0 \).
We can also get \( t_2 = t_3 \) and \( B = C \) in the same manner as in the previous cases.

The graph of function \( V_2(x) \) can be drawn as the lower polyline in Figure 5 according to Equation 8.

According to \( V_2(x) \), there are 5 pieces in the piecewise function (17) of the yarn number (linear density) \( N_{tx}(x) \).

In this function, the first two pieces and the fifth piece are the same as that in Equation 9. The functions of the third and fourth pieces are the same, but the domain is different. The graphs of all five functions are drawn in Figure 3.a. Hence the profile of slub yarn is similar to that of the function graph and can be drawn as in Figure 3.b.

The three important parameters: the length of the thickest yarn \( L_M \), the slub length \( L_s \) and yarn number (linear density) of the thickest yarn \( N_{tM} \) can be obtained from Equation 17 and Figure 3.a.

\[ L_M = 0 \quad (18) \]
\[ L_s = 2l \quad (19) \]
\[ N_{tM} = AV_2l + AB(t_2V_1)^2 \quad (20) \]

The second condition \( t_2V_1 = l \)
Like the above three conditions, the graphs of the two functions, \( V_2(x) \) and

\[ \int_{t_1}^{t_2} N_{tx}(x) \, dx = AV_2l, \quad 0 \leq x < t_1V_1 - l \]
\[ AV_2l + \frac{1}{2} AB(x + l - t_2V_1)^2 \quad t_1V_1 - l \leq x \leq t_1V_1 + t_2 - t_3V_1 - l \]
\[ AV_2l - AB(t_2V_1)x + AV_2l - AB(t_2V_1)x + AV_2l - AB(t_2V_1)x + \frac{1}{2} AB(t_2V_1)^2 \quad (t_1 + t_2)V_1 - l \leq x \leq (t_1 + t_2)V_1 \]
\[ AV_2l - AB(t_2V_1)(x - t_2V_1)(x - t_2V_1) - \frac{1}{2} AB(t_2V_1)^2 \quad (t_1 + t_2)V_1 \leq x < t_1V_1 + l \]
\[ AV_2l - AB(t_2V_1)(x - t_2V_1) + \frac{1}{2} AB(t_2V_1)^2 \quad (t_1 + t_2)V_1 \leq x < t_1V_1 + l \]
\[ AV_2l + \frac{1}{2} AC(x - t_1V_1)(x - t_2V_1) \quad t_1V_1 + l \leq x \leq (t_1 + t_2 + t_3)V_1 \]

Equation 13.
$N_{tx}(x)$, and the profile of slab yarn can be drawn as in Figures 4.a and 4.b, respectively.

The four important parameters: the length of the thickest yarn ($L_M$), the slub length ($L_s$), the yarn number (linear density) of the thickest yarn ($N_{tM}$) and length of the base yarn ($L_j$) can be obtained from Figure 4.a.

\[
L_M = 0 \quad (21)
\]
\[
L_s = 2l + (t_{22} + t_{23}) \times V_1 \quad (22)
\]
\[
N_{tM} = AV_{22} \quad (23)
\]
\[
L_j = 0 \quad (24)
\]

## Experimental validation

In order to verify the theoretical prediction, we carried out some experiments with the different conditions above. Generally the slub length of slab yarn made from the same raw materials cannot be obtained correctly by the direct observational method due to its unidentifiable boundary between the base yarn and slub. In order to avoid this problem, two strands of black polyester filament yarns (15.3 tex, Fangyuan chemical fibre Co. Ltd., Suzhou, China) were used as the base yarn, and (310 tex, fibre length = 29 mm) was fed into the drafting zone to produce the slab. A schematic diagram of the spinning process modified is shown in Figure 5.

A digital sample spinning machine (DSSP-01, Digitized Textile Technology Institute of Tianjin Polytechnic University, China) which can control every roller’s speed by means of a computer was employed to produce slab yarns using intermittent acceleration of the back-mid rollers under different conditions in the experiments. The spinning parameters are listed in Table 1. The four sets of parameters correspond to:

1# $t_{22}V_1 > l$,  
2# $t_{22}V_1 = l$,  
3# $t_{22}V_1 < l$,  
4# $t_{22}V_1 = 0$.

After spinning, the slab yarns produced were recorded by digital camera (Canon G10, Canon Co. Ltd., Japan), as shown in Figure 6 (see page 34). In order to reduce the error, each slab yarn was measured at least 20 times at different positions under each condition and the mean value was calculated. The experimental results obtained from the different spinning conditions are listed in Table 2 (see page 34).

The experimental results showed that the
slub length and that of basic yarn measured agreed well with the theoretical predictions and showed that the model has high prediction accuracy.

### Conclusions

A theoretical model was proposed to deal with the yarn count and yarn profile of the ring-spun slub yarn spinning process according to the process parameters. Spinning experiments were carried out to validate the model. The experimental results show very good agreement with those obtained from the theoretical predictions. The conclusions presented provide a general way of understanding and producing slub yarn with the profile desired in the ring-spun slub yarn spinning process.

### Acknowledgments

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### References


### Table 2

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**Figure 6.** Images of the slub yarns produced from the four experiments under different conditions, a) t22V1 > l, b) t22V1 = l, c) t22V1 < l, d) t22V1 = 0.