Methodology for Evaluation of Fabric Geometry on the Basis of the Fabric Cross-Section

Abstract
A woven fabric structure is defined by mutual threads interlacing in the fabric as well as the basic parameters of the fabric. The interrelation among fabric parameters can be obtained by considering a geometrical model of the fabric and specific experimental methods. The geometrical model is mainly concerned with the shape taken up by the yarn in the warp or weft cross-section of the fabric. This article provides a possible methodology for evaluation of geometric parameters of threads in the real longitudinal and transverse cross-sections of fabric. From an individual cross-section of the fabric using image analysis it is possible to define the diameter of threads, their deformation, thread spacing, the maximum displacement (height of binding wave) of the thread axis, the angle of the thread axis (interlacing angle), the length of the thread axis in the cross-section of the fabric, the crimp of threads in the fabric, and the real shape of the binding wave through wave coordinates. The parameters mentioned are possible to use as input for mathematical modelling of the fabric structure and for prediction of mechanical and end-use properties of fabrics.

Introduction
All the process of weaving is binding point formation. Their dimensions and tension gradually change from the cloth point formation. Their dimensions and tension in describing woven fabric behaviour is possible to evaluate changes in the interlacing which are given by the different warp as well as weft tension. In a steady state it is necessary to respect the regularity of the thread interlacing. This regularity defines the final quality of woven fabric; cross sections in this case have to be identical. Each irregularity or difference in the cross section determines the fabric fault. The structure of the woven fabric is usually defined by the weave, the material of the yarn, thread density, and yarn count [4]. These specifications determine the areal geometry of woven fabric. Areal geometry defines only some selected properties of the fabric. A significant role in describing woven fabric behaviour is played by the three-dimensional geometry. The spatial geometry is affected by the type and adjustment of the weaving loom. The fabric geometry influences the mechanical and end-use properties of fabrics such as handle, elongation, crimp, maximal density, and weight [1, 6]. The fabric structure also determines the weavability of fabrics and weaving process. To describe the dependences among the fabric structure, weaving process and resultant fabric properties, it is not possible to operate with theoretical models only. In some cases it is necessary to operate with empirical findings which have an acceptable table format or mathematical formulation. The same fabric construction (identical density, material, yarn count) woven in different conditions or on different weaving looms can have different variations in the spatial geometry of woven fabric. Empirical findings are possible to use not only for determination of the fabric structure but they also establish a basis for calculation of selected fabric properties, for a description of various changes in the fabric geometry, etc.

The foundation for a study of the areal geometry of fabric is the binding cell (the crossing of an end and pick) in a plan view [11]. The initial idea of the areal structure is a model of woven fabric which has crossing points formed in one plane. This geometry operates with hundred-per-cent cover of the fabric and with the possibility of mutual recalculation of thread densities and yarn count for the same cover factor of the fabric and weaving resistance. This creates a false impression that it is possible to weave however dense woven fabric in any combination of the warp and weft density. The thread density and its cover factor of the fabrics are limited by the possible mutual positions of the warp and weft threads in the space of the binding cell [3, 18]. The mutual positions of the threads in the woven fabric create and describe the spatial geometry of the interlacing of the woven fabric.

Geometry of woven fabric – description of basic parameters
The basic geometrical characteristics of the fabric structure of the binding cell are possible to summarise as a vector of the input geometric structural parameters [1]. This vector consists of the parameters mentioned: \( d_{\text{warp, weft}} \) (yarn diameter), \( h_{\text{warp, weft}} \) (height of the binding wave), \( e_{\text{warp, weft}} \) (relative waviness), \( D_{\text{warp, weft}} \) (thread’s sett), \( A \) (a woven distance), \( B \) (warp distance), \( L_{\text{warp, weft}} \) (length of yarn between yarn intersections), \( C_{\text{warp, weft}} \) (crimp of threads), \( t \) (thickness of a fabric). In real woven fabric it generally does not apply that the warp and weft binding points lie in the same plane [2], see balanced and unbalanced fabric in Figures 1 & 2 (see page 42).

The interlacing of one end and pick creates the binding cell of the woven fabric. The size of the binding cell is defined by the actual spacing of the weft and warp yarn. The spacing of the weft yarn presents the depth of the binding cells along the longitudinal axis and the spacing of the warp yarn presents the binding cell in the direction of the transverse axis. The
The waviness $\varepsilon_{\text{warp}}$, $\varepsilon_{\text{weft}}$, of interlacing is given by the height of the binding wave in the woven fabric. The height of the warp binding wave $h_{\text{warp}}$ and that of the weft binding wave $h_{\text{weft}}$, see Figure 3, is the maximum displacement of the thread axis normal to the plane of the woven fabric.

The height of the binding wave $h_{\text{warp}}$ and $h_{\text{weft}}$ is given by Equations 5 and 6.

$$h_{\text{warp}} = \varepsilon_{\text{warp}} d_s$$

$$h_{\text{weft}} = (1 - \varepsilon_{\text{weft}}) d_s$$

The thickness of the woven fabric $t$ is the double value of the maximum from values $h_{\text{warp}} + d_{\text{warp}}/2$ and $h_{\text{weft}} + d_{\text{weft}}/2$ [3]. We can express the thickness of the fabric by the following Equation 8.

$$t = \left( d_{\text{warp}} + d_{\text{weft}} \right) \max \left( \frac{2h_{\text{warp}}}{d_{\text{warp}} + d_{\text{weft}}} + \right)$$

$$+ \left( \frac{2h_{\text{weft}}}{d_{\text{warp}} + d_{\text{weft}}} \right) + \left( \frac{2h_{\text{warp}}}{d_{\text{warp}} + d_{\text{weft}}} \right)$$

$$+ \left( \frac{2h_{\text{weft}}}{d_{\text{warp}} + d_{\text{weft}}} \right)$$

For balanced fabric the fabric thickness given by Equation 9:

$$t = \left( d_{\text{warp}} + d_{\text{weft}} \right) \max$$

Previous geometric models result from initial geometric assumptions about yarn axes and cross-sections [2]. The central axes of the binding wave are formed only from abscissas or from ring arches and abscissas, from other curves. The yarn cross-section at the binding points of the fabric is possible to substitute by circular or other shapes [14].

The problems with the application of Peirce’s model [12] or other models are in the description of some real parameters of the woven fabric (values $d_{\text{warp}}$, $d_{\text{weft}}$, $h_{\text{warp}}$, $h_{\text{weft}}$, etc.) which usually we do not know. Estimation of these geometric parameters can be realised by using image analysis through fabric cross-section processing [15]. From the woven fabric cross-section it is possible to evaluate the shape of the binding wave and yarn deformation.

The shape of the binding wave and that of individual yarns in the woven fabric cross-section are possible to evaluate from the real woven fabric cross-section [1, 6, 7, 8, 11, 26, 28] on the basis of

- central axes of the binding wave (central axes are important for determination of the real length of the binding wave as well as for the thread crimp),
- individual coordinates of the binding wave, see Figures 4 & 5 (individual coordinates are possible to use for simulation of the thread interlacing as the limits of possible fabric densities, about the stability of the weaving etc. can be deduced from the description of mutual relations between tension and geometrical changes in the binding cell. The condition of the mathematical formulation of the binding wave and yarn cross-section for individual interlacing is given in [2]. Many attempts have been made to find a suitable model describing the binding cell, i.e. to express mathematically the shape of the binding wave in a given thread crossing in a fabric in a steady state. The Peirce model [12, 27], hyperbolic model, and sine shape are the models most used, which are related to plain weave, as is known. The model must be equally operative, and must describe the binding repeat of the threads of both sections (longitudinal and transverse sections) as well as the influence of the bending rigidity and material profile on the shape of the threads interlacing in the passage from the right side on the reverse side of the fabric and by contraries [1].

Analysis of the binding wave and yarn shape in a real fabric cross-section

All necessary information about the balance of variable forces, about the deformation of binding points (cells), about the waviness $\varepsilon_{\text{warp}}$, $\varepsilon_{\text{weft}}$, of interlacing is given by the height of the binding wave in the woven fabric. The height of the warp binding wave $h_{\text{warp}}$ and that of the weft binding wave $h_{\text{weft}}$, see Figure 3, is the maximum displacement of the thread axis normal to the plane of the woven fabric.
marked can be magnified or decreased by mouse scrolling, and thus the overlay image and binary image, respectively, of the colour woven fabric cross-section image can be precisely defined, see Figure 9.

The convex hull is applied on the binary image of the woven fabric cross-section, which is the nearest set of binary image convex curves [10], see Figure 10.

Parameters of the Minimal Feret Diameter (minimal perpendicular distance of two tangents to the object surface [10] – i.e. the distance is equal to the thickness of woven fabric in a perfectly horizontal position of threads in the fabric cross-section) and orientation (angle between the main axis of the woven fabric cross-section object and x axis in the system of NIS Elements [10], ideally equal to 0°) are evaluated. In the case of the non-zero Orientation the real thickness of the woven fabric is calculated according to

\[
\text{Thickness} = \text{MinimalFeret} \cdot \cos(\text{Orientation})
\]  

(10)

The object defined by the convex hull is eliminated from the central line – fabric axis, the coordinates of which are measured. The original overlay (binary) image of the fabric cross-section (Figure 9) is invoked from the reference level [10], the user clears cross-sections of yarns in the fabric cross-section (Figure 11 – blue circle tool), and a new overlay image of just the binding wave of the fabric cross-section is originated.

The binding wave is eliminated from the central line – the axis of the binding wave,
its coordinates together with its Length and Maximal Feret Diameter (maximal perpendicular distance of two tangents to object surface [10] – i.e. the distance of the binding wave endpoints) is measured. Then the weft or warp crimp can be calculated according to Equation 11.

\[
\text{Crimp} = \frac{\text{Length} - \text{Maximal Feret}}{\text{Maximal Feret}} \quad (11)
\]

The following parameters are measured on these binary objects:
- Equivalent Diameter – diameter of a circle with the same area as the real shape,
- Perimeter – perimeter of the real object,
- Maximal Feret Diameter – the longest distance of two tangents to the object surface [10],
- Minimal Feret Diameter – the shortest distance of two tangents to the object surface [10],
- Circularity – shape parameter, ratio between the real area of the object and the circle area, which has the same perimeter as the real object. The circular shape has a circularity equal to 1 [10],
- CenterX – x coordinate of center of gravity, CenterY – y coordinate of center of gravity. Then on the basis of the yarns’ cross-section parameters it is possible to describe the shape factor of yarn for individual yarns in the cross-section. The shape factor is calculated according to Equation 12.

\[
\text{Shape factor} = \frac{\text{Perimeter}}{\pi \cdot \text{Equivalent Diameter}} - 1 \quad (12)
\]

The same parameters are also measured for yarns’ cross-sections in the fabric cross-section defined by the convex hull, see Figure 13.

Measuring of the woven fabric geometry parameters – concrete example

This example (Figure 14) is from a series of PP fabric in plain weave. For the warp
and weft system two-ply yarn is used. The yarn count is 25 × 2 tex, the warp density - 16 ends/cm, and the weft is 9 picks/cm.

The -output of the description of the longitudinal as well as transverse cross-section are the geometric parameters of the woven fabric, the binding wave and yarns’ cross-sections mentioned in Tables 1 & 2. On the basis of individual coordinates of the binding wave and yarns’ cross-sections (Figure 15, 17) it is possible to compare the theoretical shape of interfacing with a real shape.

Verification of methodology accuracy
Geometric parameters measured on the basis of methodology mentioned above must satisfy the basic conditions of interfacing. Validation of this methodology can be done by comparing warp and weft waviness values as well as thickness values. Identical warp and weft threads are possible to display in two different fabric cross-sections.

The values of warp waviness measured from longitudinal cross-sections must be identical with those of the warp waviness measured in the transverse cross-section (the same conditions have to be valid for the weft waviness), see Figures 19 - 21 (pages 46).

The same conditions have to be valid for the thickness value. The thickness measured in the longitudinal cross-section must correspond to that measured in the transverse cross-section. The comparison of the other output geometric parameters measured is not mentioned in the paper because these parameters are dependent on the input parameters of individual cross-sections.

The results displayed are based on the box-plot, being a standard diagnostic tool. The large box contains 50% of the data; its upper edge corresponds to the 75th percentile, its lower edge to the 25th percentile. The median is located in the middle of the white rectangle inside the green box. The width of the white rectangle inside the green box corresponds to that of the confidence interval for the median. Two black lines correspond to the inner fence. The data points outside the inner fence are marked red. They might be considered as outliers.

Table 1. Fabric geometry parameters (longitudinal cross-section) defined by experimental method mentioned.

<table>
<thead>
<tr>
<th>Fabric geometry parameters</th>
<th>Parameters of binding wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>MinFeret, μm</td>
<td>MaxFeret, μm</td>
</tr>
<tr>
<td>684.647</td>
<td>166.000</td>
</tr>
</tbody>
</table>

Table 2. Fabric geometry parameters (transverse cross-section) defined by experimental method mentioned.

<table>
<thead>
<tr>
<th>Fabric geometry parameters</th>
<th>Parameters of binding wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>MinFeret, μm</td>
<td>MaxFeret, μm</td>
</tr>
<tr>
<td>744.8</td>
<td>18.0</td>
</tr>
</tbody>
</table>

Figure 19. Warp and weft waviness D2 = 9 in cm (evaluation of longitudinal and transverse cross-section).

| Conclusion |

The mechanical and end-use properties of the woven fabric are dependent on the fabric structure. The woven fabric structure is influenced by geometric parameters. Some parameters can be expressed by the mathematical models, whereas some are based on experimental methods.

On the basis of the methodology mentioned above, it is possible to define in the individual fabric cross-section the diameter of threads, their deformation, thread
spacing, maximum displacement (height of binding wave) of the thread axis, the angle of the thread axis (interlacing angle), the length of the thread axis in the cross-section of the fabric, the crimp of threads in the fabric, the real shape of the binding wave through a wave coordinate, and the fabric thickness. The main advantages of this methodology are as follows:

- it helps to establish the relationships among various geometrical woven fabric parameters,
- individual parameters are possible to use for prediction of selected woven fabric properties,
- it helps in the comparison of the real parameters and theoretical values of the woven fabric,
- on the basis of selected geometry parameters it is possible to calculate the resistance of the woven fabric to mechanical deformation, internal forces in the warp and weft given by the waviness of individual threads, etc.

This methodology is possible to use for the definition and evaluation of threads interlacing from the cloth fell in the forming zone to as far as some place of the steady state in the fabric. For each weave of the fabric and binding wave in the cross section it is characteristic that its pattern of the binding is repeated regularly (periodically) across the whole fabric width and is continuous. In the forming zone it is possible to evaluate changes in the interlacing which are given by the different warp as well as weft tension (in this case we can obtain the variable cross section as well as variable geometry of the fabric). In a steady state it is necessary to respect the regularity of thread interlacing. This regularity defines the final quality of the woven fabric; cross sections in this case have to be identical. Each irregularity or difference in the cross section determines the fabric fault.

The purpose of this paper is to introduce a possible method of real cross-section evaluation of woven fabrics which satisfy the basic conditions of interlacing for determining real values of the fabric geometry.

The aim of this paper is not a comparison of real values with the theoretical ones of known theoretical models, where the differences are given by the mathematical apparatus that was used for definition of interlacing in the cross section.

Acknowledgement

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References

4. Szosland J. Modelling the structural barrier ability of woven fabrics. Textile research, Department of Textile Architecture, Technical University of Łódź, Poland.