Bojun Xu,
Ye Zhu,
Xinjin Liu

Study on the Relationships Between Fibre Displacement and Strain by the Finite Element Method

Key Laboratory of Eco-Textile, Ministry of Education, Jiangnan University,
Wuxi 214122, P. R. China,
E-mail: micky86_2008@126.com

Abstract
The aim of this study was to establish a mathematical model of the relationship between fiber displacement and strain in the twisting process; the cross-section of yarn was taken at random. Based on the differential method, the plane stress was analysed mathematically, and stress and strain balance equations of the yarn cross-section were obtained. Then a geometry model of the cross-section was established using ANSYS10.0, which is a kind of Finite Element Analysis Software. Changes in the displacement can be simulated by this model, which reflects the relationship between the displacement and stress. The results showed that there are some relationships between the strain and displacement.

Key words: *cross-section, finite element method, fibre displacement and strain.*

Introduction

The fracture of yarn always occurs in the weakest cross-section [1]. In the twisting process prestresses are produced [2, 3], and the stresses decrease from the yarn outer layer to the inner one. In previous literature, this conclusion has been presented through specific experiments and corresponding images [4 - 7]. In this paper, this conclusion will be verified again using the Finite element method.

The finite element method was developed on the basis of physical analysis of structural mechanics and has been used effectively in many fields [8], such as computational mathematics [9, 10], computational mechanics [11] and so on. It can deal with a wide variety of physical issues such as linear elastic mechanics, non-linear stress-strain relations, fluid dynamics, etc. Especially, due to its powerful computing function, the Finite element method has been used in textile research in recent years [12 - 16].

Motivated by all these works, this paper attempts to study the relationships between fibre displacement and strain using the Finite Element Method. Firstly, the stress of the yarn cross-section was analysed mathematically, and plane stress and strain balance equations were obtained by using the differential method. Then, a geometry model of the cross-section was established using ANSYS10.0, which is regarded as one of the most professional types of software. Finally, a displacement-strain curve of the fibre and stress nephogram was obtained by computer simulation.

Theoretical analysis of fibre displacement and strain

Stress analysis

Mostly, an elastic body has an arc-shaped boundary, such as disc, cylinder, etc. For these objects, it is more suitable to use polar coordinates to describe their boundary shapes. Normal yarn can be treated as a cylindrical elastic body, and its shape of cross-section can be considered as round. At the polar coordinates, the location of any point in the plane is determined by the distance (r) from the point to the coordinate origin (O), as well as by the angle (θ) between the direction of r and the x-axis.

As shown in ***Figure 1***, in a certain cross-section of yarn, the micro-unit abcd is shaped by two arcs (ad, bc) and two radial lines (ab, cd). The distance between ad and bc is dr, and the angle between ab and cd is $d\theta$. On the micro-unit, normal stress in the direction of r is called radial stress, which is expressed by σ_r. Normal stress in the direction of θ is called tangential stress, which is expressed by σ_θ. Shear stress is expressed by $\tau_{r\theta}$ or $\tau_{\theta r}$. R denotes the radial volume force, and S stands for the circumferential volume force.

In order to obtain balance equations for the polar coordinates, two balanced relationships are set up along two directions of the coordinate axes, respectively. Because of the interaction of the internal stress and volume force, the micro-unit is in balance. Firstly, we discuss the balance of radial stresses of the yarn cross-section. As shown in ***Figure 1***, the normal stress on arc ad changes along the radial direction (r-axis at polar coordinates), and its increment from arc ad to arc bc is given as follows:

$$\sigma_r + \frac{\partial \sigma_r}{\partial r} dr \qquad (1)$$

According to the internal force equilibrium, the radial force in the yarn cross-section (that is, r-axis) is zero. Then the ***Equation 1*** can be given by ***Equation 2***.

Considering $\sin\frac{d\theta}{2} \approx \frac{d\theta}{2}$ and $\cos\frac{d\theta}{2} \approx 1$ then ***Equation 2*** can be simplified as ***Equation 3***.

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta r}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} + R = 0 \qquad (3)$$

Similarly, we can get the hoop balance equation of the yarn cross-section as follows:

$$\frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{\tau_{r\theta} + \tau_{\theta r}}{r} + S = 0 \qquad (4)$$

Hence ***Equations 3*** and ***4*** present balance equations of the stress and volume force at the polar coordinates.

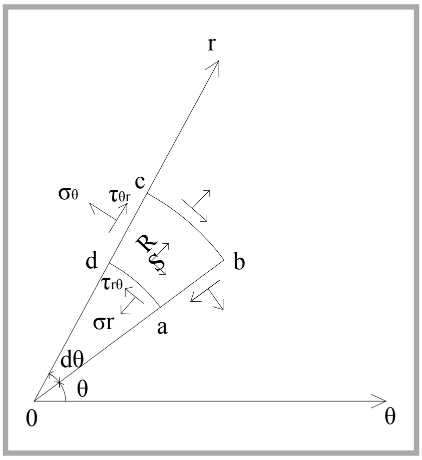

Figure 1. *Stress of cross-section.*

$$\left(\sigma_r+\frac{\partial\sigma_r}{\partial r}dr\right)(r+dr)d\theta-\sigma_r\cdot rd\theta-\left(\sigma_\theta+\frac{\partial\sigma_\theta}{\partial\sigma}d\theta\right)dr\cdot\sin\frac{d\theta}{2}-\sigma_\theta dr\cdot\sin\frac{d\theta}{2}+$$
$$+\left(\tau_{\theta r}+\frac{\partial\tau_{\theta r}}{\partial\theta}d\theta\right)dr\cdot\cos\frac{d\theta}{2}-\tau_{\theta r}\cdot dr\cdot\cos\frac{d\theta}{2}+R\cdot r\cdot d\theta dr=0 \qquad (2)$$

Equation 2.

Theoretical analysis of the displacement and strain of fibre

Generally, the path of the displacement of fibre looks like a cylindrical helix, and strain will appear in both the radial and circumferential directions. In order to establish differential equations effectively, we solve the problem of displacement in the radial direction first, and then solve that in the circumferential direction. Finally, the total strain is obtained. In this paper, the cross-section of yarn is simplified as a two-dimensional plane. In the twisting process, fibres are transferred from one layer to another, which can be interpreted as a radial displacement, or in the same layer, which is interpreted as a circumferential displacement, assuming the direction of twist is counter-clockwise.

At the polar coordinates, the displacement of each point is determined by two parts: radial displacement u_r and circumferential displacement u_θ. Correspondingly, there is radial strain ε_r and circumferential strain ε_θ..

In the twisting process, for convenience of analysis, we make the following assumption:

- **Assumption 1.** Besides circumferential displacement, there is only radial displacement.

Firstly, we will discuss the radial displacement, which is shown in ***Figure 2***. If the radial line segment (*PA*) is moved to (*P'A'*) and the circumferential line segment (*PB*) is moved to (*P'B'*), the displacements of the three points *P, A, B* are given as follows:

$$PP'=u_r,\quad AA'=u_r+\frac{\partial u_r}{\partial r}dr,\quad BB'=u_r+\frac{\partial u_r}{\partial r}dr \qquad (5)$$

Because of the spinning tension and twisting effect, the outer fibres are transferred from the external layer to the internal one, and the fibres in the center are squeezed out. According to the strain definition, partial differential expression can be presented in ***Equation 6***, which is the radial strain. In ***Figure 2***, point P is transferred to P', and point B is transferred to B', hence the length of curve PB increases, which causes an increase in yarn diameter in the actual spinning process. Now, the fibres edged out are suffering more spinning tension, the strain of which is expressed in ***Equation 7***.

$$\varepsilon'_r=\frac{P'A'-PA}{PA}=\frac{AA'-PP'}{PA}=\frac{\left(u_r+\frac{\partial u_r}{\partial r}dr\right)-u_r}{dr}=\frac{\partial u_r}{\partial r} \qquad (6)$$

$$\varepsilon'_\theta=\frac{P'B'-PB}{PB}=\frac{(r+u_r)d\theta-rd\theta}{rd\theta}=\frac{u_r}{r} \qquad (7)$$

Because the rotating angle will appear when the fibres in the yarn body move along the radial direction, the corresponding shear strain $\gamma'_{r\theta}$ is equal to the rotating angle of the circumferential, presented as follows:

$$\gamma'_{r\theta}=\frac{BB'-PP'}{PB}=\frac{\left(u_r+\frac{\partial u_r}{\partial\theta}d\theta\right)-u_r}{rd\theta}=\frac{1}{r}\frac{\partial u_r}{\partial\theta} \qquad (8)$$

Secondly we will discuss the circumferential displacement, which is shown in ***Figure 3***. If point (P) is moved to (P'') and point (B) is moved to (B'') correspondingly in the circumferential direction, the displacements of the three points P, A, B are presented as follows:

$$PP''=u_\theta,\quad AA''=u_\theta+\frac{\partial u_\theta}{\partial r}dr,\quad BB''=u_\theta+\frac{\partial u_\theta}{\partial\theta}dr \qquad (9)$$

In this paper, we only analyse the counter-clockwise displacement. The circumferential strain is presented in [10] taking into account ***Equations 8*** and ***9***. Since the displacement occurred in the same yarn layer, the radial distance is fixed, that is, the radial strain is zero, shown as (11).

$$\varepsilon''_\theta=\frac{P''B''-PB}{PB}=\frac{BB''-PP''}{PB}=\frac{\left(u_\theta+\frac{\partial u_\theta}{\partial\theta}d\theta\right)-u_\theta}{rd\theta}=\frac{1}{r}\frac{\partial u_\theta}{\partial\theta} \qquad (10)$$

$$\varepsilon''_r=0 \qquad (11)$$

As shown in ***Figure 3***, if point (P) is moved to (P'') and point B is moved to B'', the corresponding torsional angle α owing to the transfer is as presented in [12]. When the fibres are displaced in the circumferential direction, radial resistance β appears, given as $\beta=-\frac{u_\theta}{r}$. Hence, the total shear strain $\varepsilon''_{r\theta}$ generated by reversing should be expressed as follows:

$$\alpha=\frac{AA''-PP''}{PA}=\frac{\left(u_\theta+\frac{\partial u_\theta}{\partial r}dr\right)-u_\theta}{dr}=\frac{\partial u_\theta}{\partial r} \qquad (12)$$

$$\gamma''_{r\theta}=\ +\beta=\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r} \qquad (13)$$

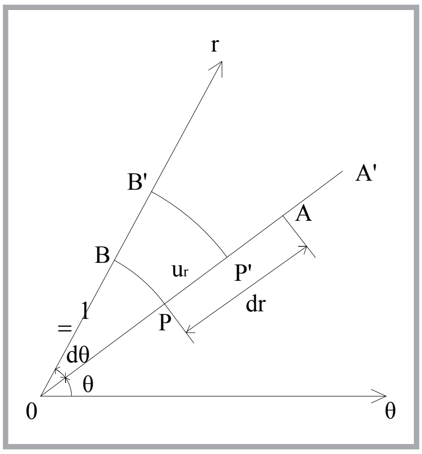

Figure 2. *Radial displacements.*

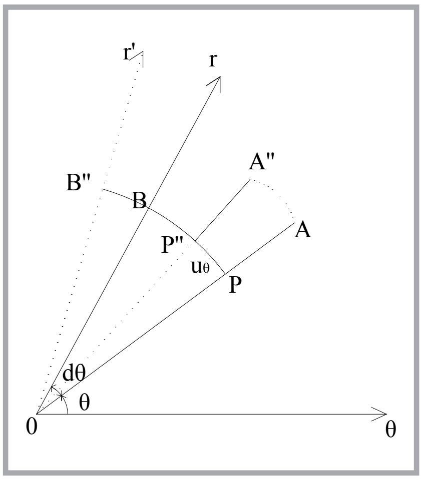

Figure 3. *Circumferential displacement.*

Finally, we obtain the relationship between the fibre displacement and strain:

$$\begin{cases} \varepsilon_r = \dfrac{\partial u_r}{\partial r} \\ \varepsilon_\theta = \dfrac{u_r}{r} + \dfrac{1}{r}\dfrac{\partial u_\theta}{\partial \theta} \\ \gamma_{r\theta} = \dfrac{\partial u_\theta}{\partial r} + \dfrac{1}{r}\dfrac{\partial u_r}{\partial \theta} - \dfrac{u_\theta}{r} \end{cases} \quad (14)$$

Simulation of displacement – strain by using ANSYS

In this section, a simulation of the relationship between the fibre displacement and strain, as presented in [14], will be given using the Finite element method. ANSYS is one of the most professional types of software and can efficiently evaluate static types of structure, dynamics and vibration, as well as solve linear and nonlinear problems. Therefore, we chose ANSYS10.0 in this paper.

Finite element model

Definition of element properties

Taking into account computational accuracy and efficiency, we chose the element "Plane2". The parameters of the yarn selected are as follows: metric number 48.6 tex, and volume weight 0.75 g/cm^3. According to the formula, we can calculate the diameter of the yarn, which is 0.28 mm.

Meshing

Because the yarn is a relatively simple physical model, we adopt free meshing. This method generally does not need to define the number and size of segments, as ANSYS will provide intelligent control. The size of the element is subject to the yarn cross-section. We set the mesh size at 0.05 mm, the number of elements - 138, and the number of nodes is 305. ***Figure 4*** shows the state after meshing.

Results

The loads include the degrees of freedom constraints, and the displacement. In this paper, the yarn cross-section will be regarded as an ideal plane, and only the outer stress and strain of the surface layer will be studied; the central axis of the yarn is assumed to be fixed. We can obtain all variations of the displacement, as shown in ***Figure 5***.

All loads are defined by the SOLU processor, and corresponding simulation parameters are selected as follows: the analysis method is static mechanical analysis, the analysis type - linear, and the solver is automatically solved. Finally, we can obtain the distribution of the stress located in the external layer, as shown in ***Figure 6***, where the shade stands for the size of the stress. From Figure 6, we can see that the stress is gradually reduced from the outer layer

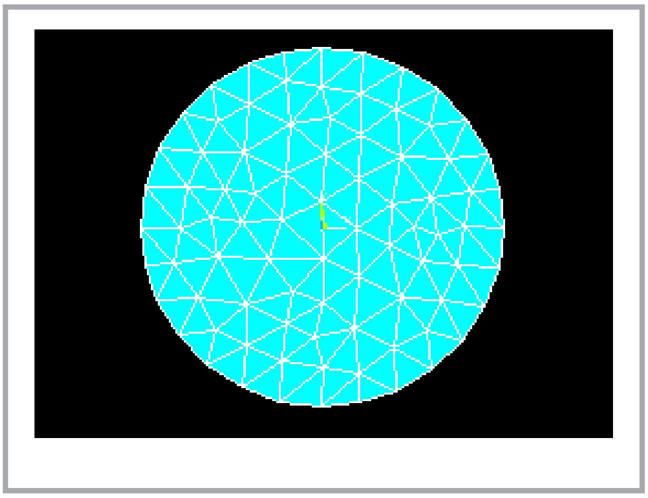

Figure 4. *Mesh of the yarn cross-section.*

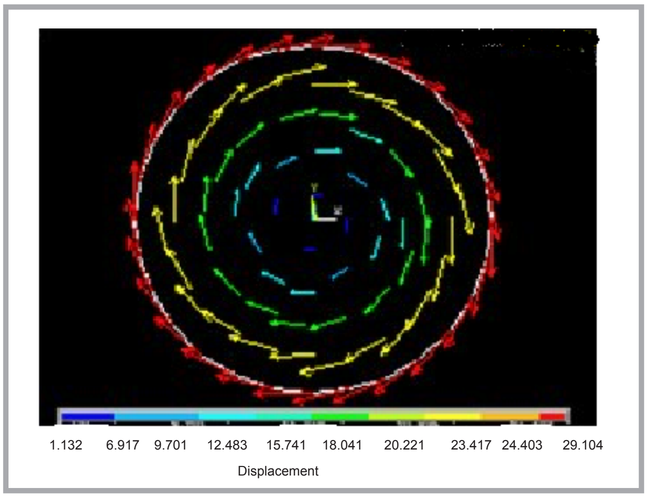

Figure 5. *Stress of the displacement of the cross-section.*

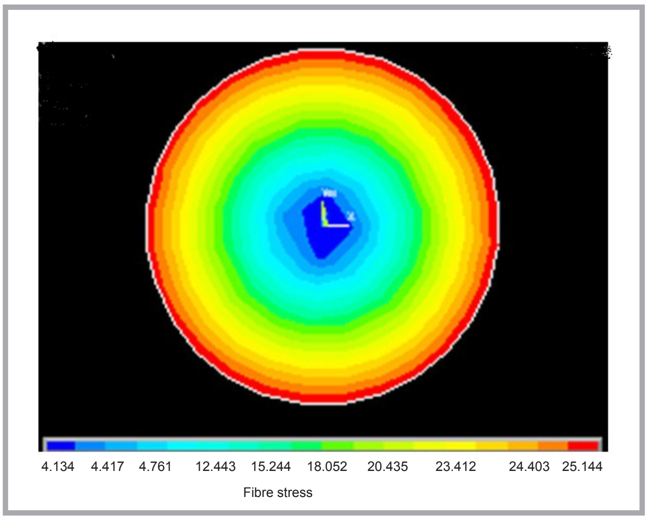

Figure 6. *Strain distribution of the yarn cross-section.*

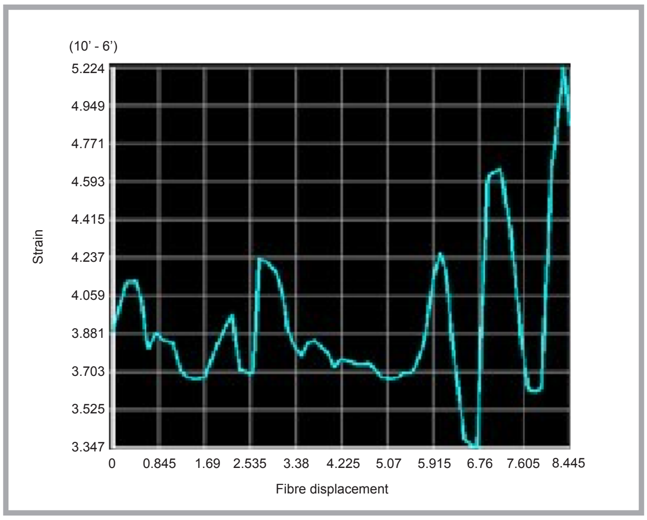

Figure 7. *Curve of displacement- strain.*

to the inner one, which is consistent with that presented in previous work.

Analysis of result

Actually, the displacement of the points can be seen in the transfer of fibres in the yarn. Generally, fibres are transferred along the path of the cylindrical helix. In order to more clearly describe the relationship of the displacement-strain of fibres, we chose some points from the inner to the outer layer, following the counter-clockwise principles. Using the mapping capabilities of the ANSYS Program, the curve of the displacement-strain is obtained, as shown in ***Figure*** *7*, where the abscissa represents Fibre Displacement, and the ordinate - the Fibre Strain. From ***Figure*** *7*, we can see that the curve is smooth in the beginning, but it suddenly decreases at a certain position, and volatilities then follow. This may be due to changes in the stress state of the different layers. When fibre is extruded , the stress load in the fibre increases, then the strain is greater than that at the origin. With this extrusion, fibres suffer gradually increasing stress, even beyond yield strength, which results in an increase in fibre fracture probability . Once the original fibre fracture occurs, another fibre fills it immediately, which leads to volatilities, as shown in ***Figure*** *7*.

Conclusion

The relationship between the displacement and strain of yarn was investigated mathematically, and corresponding simulations were presented using the Finite element method. The simulation results show that the displacement of fibres has a great connection with the stress of yarn, where stresses are gradually reduced from the outer to the inner layer of the yarn. Finally, a displacement-strain curve is obtained by using the mapping capabilities of the ANSYS Program, establishing the relationship between the displacement and strain theoretically. However, in the actual production process, there are some other factors such as the friction between fibres, which can affect the discussions. These need further study.

Acknowledgments

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