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# A Mathematical Model of Fancy Yarns' Strength. The First Model Developed in the World 


#### Abstract

The mathematical models of the strength of fancy yarns with continuous effects (plied twoand three-component yarns and loop yarn) were developed on the basis of the mechanical rules, phenomena occurring between the staple fibres in the component yarns and the final structure of fancy yarns. Three types of distribution of the length of the staple fibres (steady, trapezium and normal) and two types of fibre migration (full migration and lack of migration) were considered. Experimental verification was conducted and the larger influence of the distribution of the fibres'length than the migration of fibres on the strength of the final fancy yarns was proved. The essential influence of the component yarns'tensions during the twisting process on the structure and strength of fancy yarns was shown.


now to lead out mathematical formulas, which are a function of the properties of the staple fibres, plain component yarns and the structure of the final fancy yarns. The mathematical rules presented in this work are very complicated; however, it is possible to assess all the used parameters on the basis of metrology investigation, which showed the experimental verification of these models. The conclusions of the theoretical and experimental parts allow the assessment of which parameters of the staple fibres influence the strength of fancy yarns in an essential way. The theoretical models were led out for twoand three-component yarns and loop yarn in several aspects of distribution of length of the staple fibres (steady, trapezium and normal) and the form of staple yarns' migration in the component plain yarns. The models were led out on the basis of energy conservation law without consideration of the heat losses. The Almont friction law for staple fibres was used.

The following assumptions were taken into account:

- Yarns are incompressible,
- The lateral reactions from component yarns are considered,
- The twist of a single yarn is constant along the length and diameter of the yarn,
- Component yarns have perfect evenness of linear density,
- The density of yarns is constant along their length and diameter.

The coefficient of the shape of fancy yarns was introduced as an essential parameter that describes the structure of fancy yarns and their strength. The following scientific thesis was constructed: "The knowledge of the structure of fancy yarns and phenomena occurring during their twist-
ing and elongation allows their strength to be designed". The experiments were conducted on the basis of a 15 -element experiment plan with plying twist and different structures of fancy yarns as the inputs. Cotton 25 -tex yarns were used as the component yarns. These yarns were plied on the ring twisting machine in the same direction as the direction of the twist occurring in the component yarns.

## The structure of fancy yarns with continuous Effects

## The marl yarn structure

The simplest of the fancy effects, a marl yarn, is one in which two or three or more component yarns of the same count, twist and raw materials, but of different colours, are folded together to form a bal-


Figure 1. Two-component marl yarn; a) scheme [1]; b) view [2].


Figure 2. a) Graphical wrap yarn and the spiral line formed as a double helix by a single fibre in a wrap yarn [3]; b) the path of the filament in the ply [3]; c) the generation of the helix by the rotating vector [3].


Figure 3. The loop yarn with a sinusoidal effect a) the structure of yarn [5]; b) the line of cycloid [6].


Figure 4. The loop yarn with a boucle effect a) the structural model of the yarn [5]; b) the line of trochoid [6].


Figure 5. The structural model of snarls fancy yarn [5].
anced structure of fancy yarn. All the component yarns conduct the longitude and lateral forces in the same way. The helix lines, which construct the component yarns, have the same diameter, angle and pitch. (Figure 1, see page 9).

The vectorial equation of the helix line in a natural parameterization is as follows [1]:

$$
\begin{gather*}
x=a \cos \frac{s}{\sqrt{a^{2}+b^{2}}} e_{1}+a \sin \frac{s}{\sqrt{a^{2}+b^{2}}} e_{2}+ \\
+b \frac{s}{\sqrt{a^{2}+b^{2}}} e_{3} \tag{1}
\end{gather*}
$$

where:
a - amplitude of the spiral line,
b-spiral lead,
s - natural parameter (the length of the arc),
$\mathrm{e}-$ unit vectors.

## The spiral yarn structure

The spiral yarn is a plied yarn, which displays a characteristic smooth spiralling of one component around the other. The core yarn is distinguished as a straight line among the effect yarns, which constitute the spirals. It may be formed by one, two or more yarns being delivered at a greater rate to the twisting zone than the core yarn is supplied (Figure 2a). The higher longitude stress is conducted by the core yarn and this yarn is most susceptible to the break during elongation. However, the lateral stresses from the effect yarns are conducted to the direction of the core yarn. The second curvature of a single filament in the coordination of a component yarn is [3]:

$$
\begin{equation*}
\tau=\mathrm{d} \varphi / \mathrm{dz} \tag{2}
\end{equation*}
$$

The second curvature of a single filament in the coordination of a spiral yarn is:

$$
\begin{equation*}
\tau==\mathrm{d} \varphi / \mathrm{dz}+(1 / \mathrm{a}) \sin \alpha \cos \alpha \tag{3}
\end{equation*}
$$

where:
a-radius of the spiral yarn formed by the axis of the plied yarn,
$\alpha$ - angle between the axis " $z$ " and the spiral line formed by the axis of the plied yarn.

The twist in the component yarn occurring as the effect of the plying process is
[14] (Figures 2b, 2c)

$$
\begin{equation*}
t t=t+\frac{T}{4 \pi^{2} R^{2} T^{2}+1} \tag{4}
\end{equation*}
$$

where:
t - twist of a single yarn (before the plying process),
T - twist of plying,
R - radius of the plied yarn.

The parametric equation of the line, which consists of the filament in the transverse plane, is epihelix:

$$
\begin{gather*}
X=a \cos \varphi+r \cos (\gamma-\zeta) \\
Y=a \sin \varphi+r \sin (\gamma-\zeta) \tag{5}
\end{gather*}
$$

where: $\varphi$ - angular position of the ply axis, $\zeta$ - angular position of the filament with respect to the ply axis, $\gamma-$ initial value of $\zeta, \mathrm{r}$ - radius of the filament helix.

## The loop yarn structure

## Loop yarn with a sinusoidal effect

Lloop yarn with a sinusoidal effect is a compound yarn comprising a twisted core yarn with an effect yarn (or roving yarn) combined with a binding yarn. The effect yarn produces wavy projections on the fancy yarn surface. The effect is achieved by the differential delivery of the effect component as compared with the core yarns. On the other hand, the overfeed of the effect yarn is a little higher than the crimp of the effect yarn in the spiral yarn. This is the simplest form of loop yarn and the core yarn is not yet straight. The basic longitudinal stress is conducted by the core and binding yarns. The effect yarn is under lateral stress at the points of contact of the core, effect and binding yarns. (Figures 3a, 3b).

The line that describes the structure of the effect yarn is a cycloid (Figure 3b)

$$
\begin{equation*}
x+\sqrt{y(2 a-y)}=a \arccos \frac{a-y}{a} \tag{6}
\end{equation*}
$$

## Loop yarn with a boucle effect

Loop yarn with a boucle effect consists of a core with an effect yarn wrapped around it and overfeed so as to produce almost circular projections on its surface. The overfeed of the effect yarn should be higher than $150 \%$ and this overfeed guarantees that the length of the effect yarn is sufficiently large to create regular lock loops. The longitude stresses are carried out by the core and binding yarns; however, the transverse stresses are carried out by the effect yarn at the points of contact of the binding yarn, core yarn and effect yarn and they are higher if the number and surface of the contacts are higher. (Figures 4a, 4b).

The shape of the effect yarn we can describe by another type of cycloid line (trochoid - Figure 4b):

$$
\begin{gather*}
x=a(t-\lambda \sin t) ; \quad y=a(1-\lambda \cos t) \\
i \quad \lambda a=C_{1} M ; \quad \lambda>0 \tag{7}
\end{gather*}
$$

We can change the shape of the loops by changing the value of.

## Loop yarn with a snarls effect

Snarl yarn is based around a twisted core yarn and all the structure is wrapped by a binding yarn. It is made by a similar
method to the loop yarn, but uses as the effect a lively, high-twist yarn and a great overfeed. The overfeed of the effect yarn should be so large as to ensure the twisted structure of the loops (Figure 5).

## The coefficient of shape of fancy yarn

In this work, the coefficient of shape of fancy yarn was introduced as a very essential parameter, which describes the type of fancy yarns and their mechanical properties. The coefficient of shape of fancy yarn is proposed as a ratio of the diameter of the external line of the core yarn helix to the diameter of the external line of the effect yarn helix:

$$
\begin{equation*}
K=\frac{D_{s R 0}}{D_{s E 0}}=\frac{a_{R}}{a_{E}} \tag{8}
\end{equation*}
$$

where: $D_{s R 0}$ - the diameter of the external line of the core yarn helix, $D_{s E 0}$ - the diameter of the external line of the effect yarn helix, $a_{R}$ - the amplitude of the core yarn, $a_{E}$ - the amplitude of the effect yarn. If the coefficient of shape is smaller, the structure of the fancy yarn is more complex.

In this way, we can divide the fancy yarns with continuous effects according to the value of the coefficient of shape:

$$
\begin{align*}
& P=S_{1} \frac{T t_{1}}{T t_{p}} \frac{\partial e_{1}}{\partial e_{p}} \sec \alpha_{0}+S_{2} \frac{T t_{2}}{T t_{p}} \frac{\partial e_{2}}{\partial e_{p}} \sec \beta_{0} .  \tag{9}\\
& P=S_{k} \frac{T t_{k}}{T t_{p}} \frac{\delta e_{k}}{\delta e_{p}} \sec \alpha_{k 0}+S_{k+1} \frac{T t_{k+1}}{T t_{p}} \frac{\delta e_{k+1}}{\delta e_{p}} \sec \alpha_{(k+1) 0}+S_{k+2} \frac{T t_{k+2}}{T t_{p}} \frac{\delta e_{k+2}}{\delta e_{p}} \sec \alpha_{(k+2) 0} .  \tag{10}\\
& R_{E}=\frac{\mu_{E} \gamma_{f E} u_{E} \pi 10^{5} F_{R} T t_{R}\left(1+e_{R r}\right)\left(K D_{s E 0}-d_{R 0}\right) d_{E 0} T_{0}}{2 T t_{E}\left(D_{s E 0}-d_{E 0}\right) \sqrt{1+\left(D_{s E 0}-d_{E 0}\right)^{2} \pi^{2} T_{0}^{2}}} \sin 2 \alpha .  \tag{11}\\
& R_{R}=\frac{\mu_{R} u_{R} F_{E} T t_{E} \pi 10^{5} \gamma_{f R}\left(D_{s E 0}-d_{E 0}\right)\left(1+e_{r E}\right) d_{R 0} T_{0}}{2 T t_{R}\left(K D_{s E 0}-d_{R 0}\right) \sqrt{1+\left(K D_{s E 0}-d_{R 0}\right)^{2} \pi^{2} T_{0}^{2}}} \sin 2 \beta .  \tag{12}\\
& R_{E}=\frac{u_{E} \mu_{E} 100_{f E} \gamma_{f E} \pi d_{E 0} T_{0}}{2 T t_{E}\left(D_{s E D}-d_{E 0}\right) \sqrt{1+\left[D_{s E D}-d_{E 0}\right]^{2} \pi^{2} T_{0}^{2}}}\left(F_{R} T t_{R}\left(K D_{s E o}-d_{R 0}\right)\left(1+e_{R r}\right) \sin 2 \alpha+F_{B} T t_{B}\left(K D_{s E D}-d_{B 0}\right)\left(1+e_{B r}\right) \sin 2 \gamma\right) .  \tag{13}\\
& R_{R}=\frac{\mu_{R} u_{R} \gamma_{f R} \pi 10^{3} d_{R 0} T_{0}}{2 T t_{R}\left(K D_{s E D}-d_{R 0}\right) \sqrt{1+\left(K D_{s E 0}-d_{R 0}\right)^{2} \pi^{2} T_{0}^{2}}}\left[F_{E} T t_{E}\left(D_{s E D}-d_{E 0}\right)\left(1+e_{r E}\right) \sin 2 \beta+F_{B} T t_{B}\left(K D_{s E D}-d_{B 0}\right)\left(1+e_{B r}\right) \sin 2 \gamma\right] . \tag{14}
\end{align*}
$$

Equations 9, 10, 11, 12, 13 and 14.

$$
\begin{aligned}
& R_{E}=\frac{u_{E} \mu_{E} \pi 10^{5} \gamma_{\text {FE }} d_{E 0}\left(1+e_{p}\right)}{2 T t_{E}\left(\frac{D_{S R 0}}{K}-d_{E 0}\right) \sqrt{1+\left(\frac{D_{S R 0}}{K}-d_{E 0}\right)^{2} \pi^{2} T_{0}^{2}}}\left\{T t_{R} F_{R} \sin 2 \alpha \sin \alpha+T t_{B} F_{B} \sin 2 \gamma \sin \gamma\right\} .
\end{aligned}
$$

$$
\begin{align*}
& S_{k}=\frac{R_{k} l_{h} \cos \beta_{w k}}{T t_{j k}}\left\{1-2 \frac{l_{h}}{L_{f k}}+\frac{4}{3}\left(\frac{l_{h}}{L_{f k}}\right)^{2}\right\} . \\
& S_{k}=\frac{R_{k}\left(1-F_{s}\right) l_{h} \cos \beta_{w k}}{T t_{f k}\left(l_{\text {max }}^{2}-l_{\text {min }}^{2}\right)^{2}}\left\{l_{\text {max }}^{4}-4 l_{h} l_{\text {max }}^{3}+8\left(l_{h}^{2} l_{\text {max }}^{2}-l_{h}^{3} l_{\text {max }}\right)+\frac{48}{15} l_{h}^{4}\right\} . \tag{20}
\end{align*}
$$

$$
\begin{align*}
& \left.+\frac{4 s_{i}^{5}\left[E R F\left(\frac{L_{\kappa_{k}}}{s_{1}} \frac{2 l_{n}}{s_{i}}\right)-E R F\right.}{24 \sqrt{\pi}} \frac{L_{k}}{s_{l}}\right]  \tag{21}\\
& \frac{\delta e_{k}}{\delta e_{p}}=\frac{\cos \alpha_{k 0}}{\cos \alpha_{k}}\left[\frac{\sin \alpha_{k}}{\cos \alpha_{k}}\left(1+e_{p}\right) \frac{\delta \alpha_{k}}{\delta e_{p}}+1\right] \tag{22}
\end{align*}
$$

Equations 15, 16, 17, 18, 19, 20, 21 and 22.

- Merle yarn: $\mathrm{K}=1$,

Spiral yarn: $K=\frac{d_{R 0}}{D_{S E 0}}$, where: $d_{R 0}$ - diameter of the core yarn,

Loop yarn: $K \triangleleft \frac{d_{R 0}}{D_{s E 0}}$

- for loop yarn with a sinusoidal effect $K \leq \frac{d_{R 0}}{D_{s E 0}}$
- for loop yarn with a boucle effect,

K is much smaller than $\frac{d_{R 0}}{D_{s E 0}}$

- for loop yarn with a snarls effects, K is the smallest.


## Mathematical models of strength

## The strength of fancy yarn

The strength of two-component yarn is determined by Equation (9)
where: $S_{1}$ and $S_{2}$ - tension of the component yarns at the point of breaking, $T t_{1}$, $T t_{2}$ and $T t_{p}$ - linear densities of the component yarns and linear density of the final fancy yarn, $\frac{\partial e_{1}}{\partial e_{p}}$ and $\frac{\partial e_{2}}{\partial e_{p}}$ - differential of elongation of the component yarns in the function of elongation of the final fancy yarn at the point of breaking, $\sec \alpha_{0}$ and $\beta_{0}$ - secants of the angles of helix lines of the component yarns under the initial tension.

The strength of three-component yarn is determined by Equation (10).

The frictionforces occurring in the component yarn with consideration of the transverse forces acting from another component yarn in fancy yarn are described for different kinds of yarns by the set of Equations 11-18.

Equation 11 - two-component effect yarn
(designations: $\mu_{e}-$ coefficient of friction of fibres in the effect yarn, $\gamma_{f E}$ - density of fibres in the effect yarn, $u_{e}-$ diameter of fibres in the effect yarn, $F_{R}$ - tension of the core yarn, $e_{R r}$ - relative change of diameter of the core yarn during the process of elongation of fancy yarn, $T_{0}$ - twist of plying under the initial tension).;

Equation 12 - two-component core yarn; Equation 13 - three-component effect yarn; Equation 14 - three-component core yarn; Equation 15 - three-component binding yarn; Equation 16 - loop effect yarn; Equation 17 - loop core yarn; Equation 18 - loop binding yarn.

## The normal stress acting on compnent yarns

For steady distribution of fibres' length the normal stress acting on component yarns is described for steady distribution of lengthby by Equation (19),
where: $l_{h}$ - length of slippage of staple fibres in the component yarns, $\beta_{w k}-\cos$ of the angle of the fibres in the component yarns, $T t_{f k}$ - linear density of the fibres in the component yarns, $L_{f k}$ - length of the fibres;
for trapezium distribution of fibres' length, by Equation (20),


Figure 6. Strength vs twist for two-compnent spiral yarn; with: steady fibre distribution ( $a, b$ ); trapezoidum fibre distribution ( $c, d$ ), lack of fiber migration ( $a, d$ ), full fiber migration (b, c); $\longrightarrow$ Theoretical, - Experimental.


Figure 7. Strength vs. twist for tree-component spiral yarn; with steady fibre distribution ( $a, b$ ), trapezoidal fibre distribution ( $c, d$ ), lack fiber migration ( $b, d$ ), full fiber migration (a, c); $\rightarrow$ Theoretical, - Experimental.


Figure 8. Loop yarns with a sinusoidal effect; with steady fibre distribution (a, b); with trapezoidal fibre distribution ( $c, d$ ); lack fiber migration ( $a, c$ ); full fiber migration ( $b, d$ ); $\rightarrow$ Theoretical, - - Experimental.
where: $F_{s}-\%$ of fibres of which the length is smaller than $l_{\text {min }}, l_{\max }$ and $l_{\min }$ maximum and minimum lengths of fibres in the component yarn, and for normal distribution of fibres' length by Equation (21).

## Partical derivative of elongation

The partial derivative of elongation of the component yarns versus the elongation of fancy yarn is described by Equation (22).

## Comparison of the theoretical and experimental results

The comparison of tensile strength as a function of twist obtained theoretically and by experiments were caried out for two-components spiral yarn (Figure 6, see page 13), three-components spiral yarn (Figure 7) and loop yarn with sinusoidal effect (Figure 8). All these three cases of fancy yarn were tested at steady and trapezoidal staple fibre distribution, as well as with the lack of migration and with full migration of fibres.

It should be emphasised that the majority of these graphs have a similar shape, and indicate only small differences between mathematical simulation and experimental results. An essential difference occurs only between loop yarn and the two- and three component yarns. The strength of loop yarn decreases when twist the increases, whereas the strength of two- and three- component yarn increase with an increase in twist due to the same twist direction such in single component yarn. Contrary, the re-twisting process of loop yarn was carried out in the direction opposite to that in which the component yarns were twisted, according to the practice manufacturing of loop yarn on ring twisting machines.

Minimum average differences between theoretical results of strength generated by the models and results of experiments carried out, occurred in the case of an assumed trapezoidal distribution of the fibre length and full migration of fibers in the component yarns. The smallest differences occurred in the case of loop yarn. These results were not statistically significant on the basis of tStudent test with a 0.05 significance level. The highest differences occurred for the two-component plain yarn with the smallest value of twist of 100 t.p.m. A visual computer analysis of the structure
of these yarns showed that the component yarns and staple fibers were placed at a very small angle to the longitudinal axis of final yarn ( 7 degrees). During the stretch process of final yarns, this angle decreased to zero, and in precesses proceeding the diameter of yarn decreased very quickly. The slipping process of fibers in the component yarns was a decisive factor of the final yarn destruction. The changes in relation to the single component yarn played a more important role than the changes in the structure of final plied yarn. An other case of destruction of the final yarn occurred in three-plied yarns with high twist, and more important were the phenomena in relation to the final plied yarn than the forces acting on the single component yarn. The influence of the type of distribution of staple fibre length on the strength of the final fancy yarns is higher as the influence of the type of fibre migration. The reduction of the strength of fancy loop yarns with the increase in the binding twist in opposite direction was stated. In this case the binding yarn brokes the first. In the case of two- and three- component yarns all component yarns broke at the same time.

## Conclusions

1. Mathematical models that include in the interaction the full migration of fibres and the trapezium distribution of fibres' length describe the real strength of fancy yarns in the best way.
2. It was affirmed that the proper choice of fibres' length distribution is more important than including the migration of fibres in a single-component yarn.
3. It was affirmed that the stress of component yarns, friction forces in the component yarns, coefficient of shape, twist and linear densities of the component yarns influence the strength of fancy yarns in an essential way.
4. The strength of loop yarn is decreased with an increase of the twist.
5. The coefficient of shape of fancy yarns is a parameter that describes the type of fancy yarns with continuous effects.
6. The effectiveness of these models proved in the experimental way ensures the possibility of modelling the structure and phenomena occurring in
elongated fancy yarns. This is a tool to aid the knowledge about fancy yarns.

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